PHILOSOPHICAL

TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCXXVII.

PART I.

LONDON:

PRINTED BY W. NICOL, SUCCESSOR TO W. BULMER AND CO.
CLEVELAND-ROW, ST. JAMES'S;
AND SOLD BY G. AND W. NICOL, PALL-MALL, PRINTERS TO THE
ROYAL SOCIETY.

MDCCCXXVII.

ADVERTISEMENT.

THE Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions*, take this opportunity to acquaint the Public, that it fully appears, as well from the council-books and journals of the Society, as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries, till the Forty-seventh Volume: the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions;* which was accordingly done upon the 26th of March, 1752. And the grounds of their choice are, and will continue to

27699

be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgment of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body, upon any subject, either of Nature or Art, that comes before them. And therefore the thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report, and even to certify in the public news-papers, that they have met with the highest applause and approbation. And therefore it is hoped, that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

CONTENTS.

I. *Description of a Percussion Shell, to be fired horizontally from a common gun. By Lieutenant Colonel* MILLER, *late of the Rifle Brigade, and now unattached. Communicated by* R. I. MURCHISON, *Esq. F. R. S.* - - page 1

II. *On the relative powers of various metallic substances as conductors of electricity. By Mr.* WILLIAM SNOW HARRIS, *of Plymouth, Surgeon. Communicated by* J. KNOWLES, *Esq. F. R. S. November* 14, 1826. - - 18

III. *On the expediency of assigning specific names to all such functions of simple elements as represent definite physical properties; with the suggestion of a new term in mechanics; illustrated by an investigation of the machine moved by recoil, and also by some observations on the Steam Engine. By* DAVIES GILBERT, *Esq. M. P. V. P. R. S. &c.* - - 25

IV. *The Croonian Lecture for* 1826. *By Sir* EVERARD HOME, *Bart. V. P. R. S.* - - - - 39

V. *On a newly discovered genus of Serpentiform Fishes. By* I. HARWOOD, *M. D. F. L. S. Professor of Natural History in the Royal Institution of Great Britain. Communicated by* DANIEL MOORE, *Esq. F. R. S.* - - 49

VI. *An examination into the structure of the cells of the human lungs; with a view to ascertain the office they perform in respiration. By Sir* EVERARD HOME, *Bart. V. P. R. S. Illustrated by microscopical drawings from the pencil of* F. BAUER, *Esq. F. R. S.* - - - - 58

VII. *Remarks on a correction of the solar tables required by Mr.* SOUTH's *observations. By* G. B. AIRY, *Esq. M. A. Fellow of Trinity College, Cambridge, and Lucasian Professor of Mathematics in the University of Cambridge. Communicated by Dr.* YOUNG, *F. R. S. &c.* - - page 65

VIII. *On the mutual action of the particles of magnetic bodies, and on the law of variation of the magnetic forces generated at different distances during rotation. By* S. H. CHRISTIE, *Esq. M. A. F. R. S.* - - - - 71

IX. *Corrections to the reductions of Lieutenant* FOSTER's *Observations on Atmospherical Refractions at Port Bowen; with Addenda to the Table of Magnetic Intensities at the same place. By Lieutenant* HENRY FOSTER, *R. N. F. R. S.* 122

X. *Correction of an error in a Paper published in the Philosophical Transactions, entitled, "On the Parallax of the fixed Stars." By* J. F. W. HERSCHEL, *Esq. F. R. S.* - 126

ERRATA, PART IV. 1826.

Page 4, line 1, *for* "9 ten-thousandths," *read* "9 thousandths."

In Plate VI. page 189, *insert* the letter "N" at the left hand extremity of the horizontal diameter in Fig. 4; and at the upper extremity of the vertical diameter in Fig. 5, *insert* the letter "e."

Directions to the Binder.

Plates I. II. and III. should face page 124, *instead of* 174.

THE Gold and Silver Medals, founded on the Donation of BENJAMIN Count RUMFORD, for the most important discovery, or useful improvement, made and published in any part of Europe during the two preceding years, on Light, or on Heat, were awarded by the Council, held Feb. 8th, 1827, to M. FRESNEL, for his developement of the undulatory theory as applied to the phenomena of polarised light, and for his various important discoveries in Physical Optics.

PHILOSOPHICAL TRANSACTIONS.

I. *Description of a Percussion Shell, to be fired horizontally from a common gun. By Lieutenant Colonel* MILLER, *late of the Rifle Brigade, and now unattached. Communicated by* R. I. MURCHISON, *Esq. F. R. S.*

Read November 16 and 23, 1826.

BEFORE proceeding to describe this shell, it may be necessary to say a few words on the theory of rifles, with which its construction is intimately connected.

It is a principle now universally admitted, that the irregularity in the flight of shot arises from the inequality in the specific gravities of their sides, and the action of the air upon the inequalities of their surfaces. These imperfections it is impossible to guard against in casting; for although the mould may be constructed with mathematical exactness, yet the metal which is poured into it is known to occupy a larger space when hot than when cold, and is found by experience generally to contract unequally, as the process of cooling proceeds. The same mould will accordingly be found to cast shot of different sizes, and hardly ever a perfect sphere.

In a ball fired from a rifle, these imperfections are corrected by the spiral or rotatory motion of the ball round its axis, by which means any inequalities that may exist in its sides are continually shifting round the axis during the flight, and the course of the projectile is continued straight forward.

The spiral motion itself has hitherto been generally supposed to be communicated to a rifle ball, by the grooves which are made in the barrel of the gun. The following reasons, however, incline me to think that the spiral motion is given to the ball, not only by the direction it receives from the grooves of the barrel at the moment of discharge, but also by the action of the air upon the indentations in the ball itself, which the grooves of the barrel have made, when the ball is pressed against them in loading.

1. The grooves of a rifle run obliquely, making from $\frac{1}{4}$ turn or twist, to $1\frac{1}{4}$ in the length of the barrel. The diameter of the ball being somewhat greater than that of the bore, the ball requires to be driven down with considerable force, following the grooves, and receiving their impression as it descends. The ball therefore is a male, and the barrel a female screw, and unless the ball receive the impression of the grooves, it never acquires the spiral motion. This is ascertained by firing balls into a bank of hard earth, so as to flatten them a little. Unless the indentations made by the grooves of the rifle are distinctly perceptible round the edges of the balls, proving them to have struck end foremost, they never hit the mark with the necessary accuracy. If the balls are too small, and it is attempted to remedy this defect by using very thick patches, however tight they may be rammed down, they never answer the purpose intended.

2. When a rifle is fired at a moderate range, say from 100 to 300 yards, it is found that the accuracy of fire is maintained without any perceptible diminution; from which it may be inferred, that the spiral motion is maintained undiminished also. If, again, a rifle is fired at an elevation producing its greatest range, which is about $37\frac{1}{2}°$, its fire will not be so accurate as that of a plain barrel. The reason I conceive to be, that when the ball is at the summit of its flight, where it changes the direction of its course the most rapidly, the grooves have not sufficient length to enable the air to act upon them while the ball is forming the curve, so as to keep the axis and line of flight in one, by which means the position of the ball is suddenly changed, the spiral motion lost, and it continues to fly afterwards with great irregularity.

3. According to the calculation made by Mr. ROBINS, vol. i. p. 133, ed. 1761, the resistance of the air to a shot passing through it with a velocity of 1550 feet in a second, is equal to 120 times the weight of the shot. Dr. ROBISON, when treating of projectiles, in his Elements of Natural Philosophy, supposes that this resistance may be increased to 138 times the weight of the shot. But supposing it to be much below what it has been estimated at by either of those gentlemen, it is difficult to believe that a quarter of a revolution in the barrel can communicate to the ball a rotatory motion, which it has to maintain for hundreds of yards against this enormous pressure. And as this pressure falls obliquely on the grooves of the ball, I believe it to be the sustaining cause of the spiral motion.

4. So easy is it to communicate the spiral motion, that a grooved leaden ball will acquire it, in falling perpendicularly

through a roll of paper, or a gun barrel, merely from its own weight pressing against the air, so as to be distinctly visible to the eye. If the bottom of the tube is closed to prevent the air from escaping, it will be easily ascertained when the ball possesses the spiral motion, and when it does not do so, from the peculiar sound it makes against the sides of the tube.

5. If it is asserted, that the spiral motion in a rifle ball is sustained *solely* by the impulse it receives from the barrel, I am aware of no proof that can be produced in support of that opinion, either from analogy or experiment. If, on the other hand, we suppose it to be a motion, both *produced* and sustained by the action of the air, it will be in strict accordance with the effect the air is known to produce in all similar cases. Among other familiar instances may be given that of the arrow, the windmill, the shuttlecock, the smoke-jack, a window ventilator, and a grooved paper cylinder drawn through the air. And it may be observed of the arrow, shuttlecock, and cylinder, that each revolves in equal distances of air, whatever their velocity may be, and the spiral motion is maintained undiminished to the end of the flight.

Having been led to infer from the above-mentioned reasons, that the spiral motion is in all cases, both produced and sustained by the action of the air; and believing it to be impossible to reconcile theory and experiment upon any other principle, the next idea that suggested itself, was the possibility of giving the spiral motion to grooved shot, when fired from a plain barrel. The same thought appears to have occurred to Mr. ROBINS sometime before his death, about the year 1745; for, in a memorandum found among his papers,

he observes, when speaking of rifles, and the rotatory motion of the ball, " I have made some experiments on *simpler* methods of performing this, and applicable to iron bullets. My success as yet has not been what I could wish, but it has however been sufficient to encourage a farther prosecution, which, if I shall ever pursue farther, I know not." vol. i. p. 317. That eminent mathematician has left us no clue to discover what his plan was; but in his tracts, he dwells so much on the effects which the resistance of the air is capable of producing on the flight of shot, that one can hardly fail to be impressed with the idea, that it must have been in some way or other connected with that medium.

I shall now proceed to give a brief account of the experiments that have been made for the purpose of attaining the object in view. The first difficulty which presented itself, was that of applying the principle to shot of a spherical figure. The utter hopelessness of getting the air to act in the way desired, upon shot of this shape, soon led me to prefer the cylindrical to the spherical form, and it is to the improvement of that figure of shot accordingly, my subsequent endeavours have been directed.

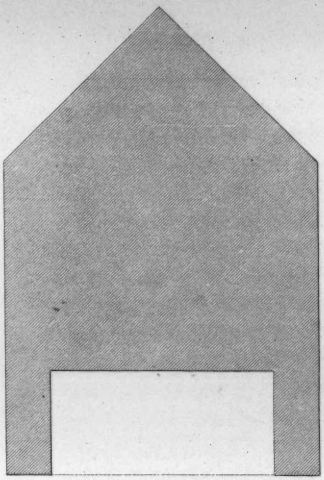
My first experiments were made merely for amusement, for the purpose of shooting seals and sea-birds, in Bantry Bay, during the summer of 1821. Though not very successful, they were repeated from time to time as opportunity offered.

Hemispherical ends to the balls were thought of, and afterwards abandoned, as it was found desirable to dispose of the weight of the ball in such a manner as to give the greatest possible length to the sides. Grooves of various kinds were

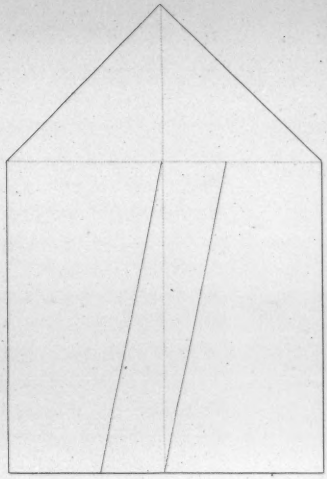
tried, and narrow ones were found not to answer. They must be of sufficient width to expose their sides, upon which the air presses, to its action throughout their length. For, if the air has not a free passage through them (Plate I. fig. 2) it will act as backwater on a mill-wheel. It is upon this principle that the blades of a windmill will not act if placed against a wall; and that rifles with very narrow grooves are found not to answer. In the beginning of 1822, some further experiments were made at Woolwich. On the first occasion, 10 grooved leaden musket balls were fired from a plain barrel through a target, at the distance of 100 yards. The balls were well finished, but heavier behind than before; and from the marks left by them in the target in passing through it, two of them appeared to have turned in their flight. From the circular holes left in the target by the others, they appeared to have passed through without turning. In the next experiment, balls of a somewhat better construction were employed. Several were fired at 40 yards, into a mass of boards and clay prepared for the purpose; and out of the number so fired, three or four were found in the exact position in which they lodged, all point foremost.

A grooved wooden shot was then fired six times from a $5\frac{1}{2}$ inch howitzer, against a bank, at 50 yards. This experiment distinctly pointed out the necessity of balancing the two extremities, as that which was heaviest showed an evident tendency to drop lower and lower during the flight.

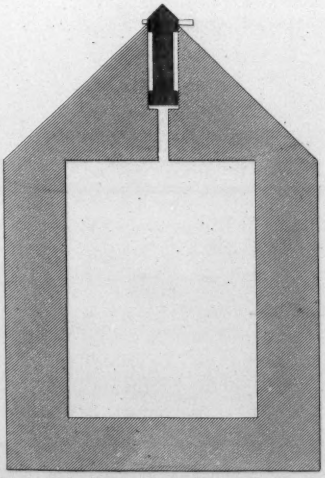
A wooden shot of the same form and proportions was then fired from the same gun, with a 3 lb. iron shot in its centre. This being properly balanced flew very steadily, hit the target, and then split without passing through it, but leaving



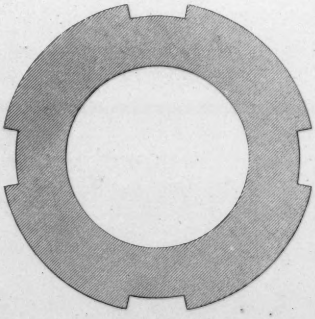
1. Vertical Section of Shot.



2. Obliquity of Grooves.



3. Vertical Section of Shell.



4. Horizontal Section of Shell.

a circular hole in the target, and a deep indentation on the cone, where it came in contact with the target.

During the following summer of 1823, I made several hundreds of experiments with grooved leaden musket-balls, fired from a plain barrel, at all ranges. When constructed with sufficient accuracy, they were found to fire very true, and to strike invariably point foremost. In order to balance them, the content of the cone, that of the cavity, and that of the part surrounding the cavity, must all three be equal, and the depth of the cavity must be equal to two-thirds of the height of the cone (Plate I. fig. 1.) The experiments with respect to twist, were found to correspond with those of rifles; that is, the velocity of the ball was found to diminish, in proportion as the twist was increased. In some of the balls used, the obliquity of the grooves was carried as high as a turn in 6 inches, which was found to diminish the velocity so much, that the person marking the shots repeatedly supposed the ball to have passed, before it reached the target. Numerous experiments were made during the same year, with wooden shot from a 24 pounder at the fort of Kinsale. They were found to range 400 and 500 yards; the largeness of their size rendered their position and flight easily perceptible to the eye; any imperfection in their construction was easily detected; and by firing them into the sea, each shot bore to be discharged a great many times before it became unserviceable. Others were afterwards fired by night with lighted fuses fixed in their sides, in order to mark their revolutions with greater distinctness. It was found that they possessed the spiral motion, and that it was maintained undiminished throughout their flight.

Having, as I conceived, conclusively ascertained by experiment, that the spiral motion may be communicated to a grooved cylindrical ball fired from a plain barrel, it occurred to me, that by the help of this principle, shells might be so constructed as to explode by percussion. The plan I have adopted for this purpose will easily be understood by an examination of Plate I. fig. 3. A round peg is placed in the apex of the cone, working in a cylinder, and a pellet of percussion powder is placed under the peg, and over the vent which communicates with the cavity of the shell. By this means, as soon as the point of the peg strikes against any hard substance, it slides in and ignites the percussion powder, which instantly communicates with the bursting charge. The cross pin is employed merely by way of precaution, and is removed as soon as the shell is put into the muzzle of the gun, which is then rammed home with a hollow-headed ramrod.

The first four shells of this construction which were tried, were made partly of iron, but principally of wood, and fired from a 24 pounder against one of the bastions of Kinsale fort, at somewhat more than 100 yards. All of them exploded upon coming in contact with the fort, the splinters flying to a great height in the air. One exploded against a rock at the distance of about 200 yards, and several, fired at a longer range, missed the object. Experiments were afterwards made with iron shells, fired from a 6 pounder, and the first were by no means successful. The shells were cast a great deal too thin, so that a considerable proportion of them exploded in the gun, and we did not succeed once in hitting a target at 240 yards. It may here be noticed as a circumstance well known, that shells are apt to explode in firing, either from

being too thin, or from a portion of sand remaining in the chamber after casting, which produces ignition from the concussion of the powder. Notwithstanding all the care that can be taken, this sometimes occurs in action; but as the propelling charge is much greater than the bursting one, the splinters are carried forward, without injury to the gunners.

The second day's experiments, made from the same gun, were more successful. The shells weighed 7 lbs. 2 oz., and the bursting charge was five oz. Eight were fired at a wooden target 8 inches thick, placed immediately in front of a soft slate rock. Several were fired through the target at 60 yards. Seven out of the eight succeeded. The eighth burst in firing. On examining the result, it was found that the explosions had produced a much greater effect upon the rock than upon the target. The target was then placed in an open situation near the sea, and two shells fired through it. It was then observed that the explosions fell behind it, and the splinters were carried several hundred yards into the sea. One shell fired into the sea with 14 oz. of powder, and at an elevation of 6°, appeared to range about 1100 yards.

In the course of the third day's experiments, several still exploded in firing. One however succeeded very well against a thicker target, and two, fired across the bay, exploded against the old castle, one at 800, and the other at 850 yards.

The next experiments were carried on at Leith Fort in the end of 1823. The shells used weighed 10 lbs., the bursting charge 7 oz., and were fired from a 9 pounder, with a charge of 1lb. Two casks filled with sand and stones, and about 4 feet each in diameter, were placed on the sea shore at low-water mark, and a target 4 feet square, and 15 inches thick,

made of old ship timber, was placed between them, and backed with sand and stones. Four shells were fired at this, at 180 yards; two of them missed, and the third exploded in one of the casks, the splinters passing out at the other side, and carrying part of the contents of the cask along with them. The fourth struck the target in the centre, and blew it to pieces, leaving an opening in it upwards of 2 feet in diameter. Several of the planks of which it was made were cut asunder, and one of them was blown backwards 12 or 15 feet.

In the next day's experiments, a target was placed at the same range, but being made of decayed timber, and backed with sand only, the effect was by no means so perceptible as on the day preceding. Several exploded between the target and the sand, without doing much execution. Four passed over the target, and ricochéd along the water for several hundred yards. It was observed that they kept circling to the right, in consequence of the grooves running in that direction. One was found in the sand under the target, cone foremost, but without the peg; and another exploded about 20 yards from the gun. From this circumstance, the necessity of adopting some contrivance for keeping the pegs firm in their position became apparent, and also of making them air-tight, in order that the compressed air might not produce ignition. The range was this day tried against that of a round shot fired from the same gun, and the two were reckoned by the artillery officers present to be as nearly as possible equal, $1\frac{1}{2}$ lb. of good powder giving the same range both with shot and shell, as 3 lbs. of that used by the Ordnance.

In the third day's experiments, the shells used were half an inch longer in the sides, and the twist reduced from 48 to 55 inches. The weight of each was 10½ lbs. including the bursting charge of 7½ oz., and they were fired with a charge of 1½ lb. The first was fired into the sea at 15° elevation, and as far as could be judged from the distance of the ships at anchor in the roads, ranged about 2200 yards. The next was fired at the Martello tower in the sea, at 6° elevation, and at a range of 1200 yards. It ricochéd from the water at about 1000 yards, and exploded against the tower. A two-feet thick target was then placed on the shore at about 110 yards, and 5 shells were fired at it. Three of them burst in it, and it was found that they had penetrated about 16 inches before the explosion took effect, and the splinters, after tearing away the back planks, passed into the sea like grape. Their range was then tried on Leith Sands, from a brass field piece. An empty shell weighing 9½ lbs, with a charge of 1½ lb. at 5° elevation, went 1330 yards: another weighing 10½ lbs. went only 1200 yards. It may be observed, that the cone of the former was sharper than that of the latter. Another weighing 10½ lbs. at 11° elevation, went 1820 yards. No wadding was used in any of the experiments either at Leith or Kinsale.

Some further experiments were made at Woolwich in April 1826, on the hull of a 28 gun ship. Part of the shells used were of the same construction as those last tried at Leith, the angle formed by the sides of the cone being 112°. In others it was raised to a right angle, which increased the length of the shell about half an inch. Their weight was about 11lbs. including the bursting charge of 8½ oz. A pasteboard wad, ⅓ of an inch thick, was put over the powder,

and made to fit the gun with great exactness, so as to prevent the escape of any part of the charge by the windage. The ship was moored in the river with her broadside to the gun, which was a brass 9 pounder, placed on the shore, at a range of 330 yards. Ten shells were fired from this position, with a charge of 1½lb. Two struck the vessel without exploding; one of them passed through both its sides; the other was found on board, without the peg, and with a piece of wood jammed into the place the peg had occupied. This circumstance leaves no doubt that the peg had, in this case, come out in firing, and renders it probable that the same accident had occurred to the other also. One exploded in firing, and another went over the vessel; two struck the water, and afterwards lodged in the ship without exploding; the remaining four exploded *upon striking her.*

In the second day's experiments, the pegs were fixed more securely by having worsted twisted round them, so as to require a blow equal to a weight of 30 lbs. falling from a height of 3 inches, to produce ignition, and a slip of paper was pasted over each, so as effectually to prevent them from slipping out when the shell was discharged from the gun. The charge was this day increased to 1¾lb. Thirteen rounds were fired at 330 yards: the gun was then retired to 450 yards, and 8 rounds more fired. Out of the 21 fired, 7 were cast so large that they stuck in the gun, and could not be rammed home. From these little could be expected, although two of them succeeded. One exploded in firing, and another on striking the water about 200 yards from the gun. One struck the water very near the ship, and exploded upon passing through the ship's side, a little above the water

mark. Six struck the vessel, all of which exploded; the others either went too high or too low, and the whole of those that went too low, rose again from the water and lodged in the ship. Upon examining the vessel, it was found that the shells had passed through her side before the explosion took effect, and that the splinters had done more than usual execution. They had gone through a cast iron caboose, torn up part of the deck, thrown down great part of a wooden partition that ran along between decks, and some of them had lodged in the opposite side of the ship. One shell cut a strong chain cable in two, which was put round the vessel for the purpose of raising her, in the event of her being sunk, some of the splinters flying backwards on explosion, while the remainder passed through her side. Another fired from the farthest range, passed through the mainmast close to the deck, and set it on fire towards the centre, so as to render it necessary to cut part of it away before the fire could be extinguished.

In the third and last day's experiments, 10 of the longer shells were fired empty, at a very large target placed against the practice bank, from a brass gun, at a range of 800 yards, with a charge of $1\frac{3}{4}$ lb. of powder. The two first were fired at 3° elevation, and passed a great way over the bank. The remaining 8 were fired at $2\frac{3}{4}$° elevation; two of those also went over, 1 grazed short, 2 passed through the target, one of them after grazing 2 yards short, and the other 3 struck the bank not far from the target; the range therefore of the shells here used, when filled and fired at 3° elevation, may be about 1300 yards. One dug out of the bank appeared to have turned; two others, on being dug out, were found

cone foremost, and another that passed through the target, left a hole perfectly circular, evidently showing that it had passed through horizontally. Four of the shorter shells were fired at 600 yards, but were found very inferior to the longer ones, both in range and accuracy of fire.

Having detailed the experiments that have been made with percussion shells, as accurately as I can, and as fully as seems to be necessary, I shall now conclude with some observations connected with the subject.

So far as range and efficiency are concerned, the experiments have perhaps been as successful as could have been expected, in so novel an invention. With respect to accuracy of fire, I am fully sensible that much still remains to be done; and to those who ask why this most important object was not more completely attained before the discovery was submitted to the public, I beg leave to answer, that no invention in gunnery, so far as I am aware, either in former or more recent times, has ever been brought to perfection without the help of long continued and laborious experiments, which from their nature are so expensive, that they cannot be expected to be prosecuted at the cost of any individual. In the present instance, only one hundred and four shells have been fired altogether, eighty-five of which were filled with powder, and out of these, thirty-nine exploded upon striking the objects fired at. In the experiments made at Woolwich, on the Pheasant sloop of war, in the river, which are certainly the most important that have yet been made, only eleven succeeded out of thirty-one; no great number certainly, but at the same time enough to have destroyed the vessel, had they been heavy metal. The fire at present is sufficiently accurate

for the range at which naval actions are generally fought, but the object in view is to make them available to the full extent of their range, and I shall accordingly point out the means by which I conceive this object may be very much facilitated. Considerable improvement may, I think, yet be made in the mode of casting them; for although the gentlemen of the Carron Company have bestowed great pains on those that were cast by them, it seldom happens in matters of this kind, that the most simple process is discovered in the first instance. They may also be turned in a lathe, by means of machinery, which their shape will allow them to be with great facility, and thus rendered perfectly cylindrical, and of the same size. In the course of the experiments that have been made, the shell has also been greatly improved by an addition to the length originally given to it. This might, *a priori*, have been expected, as its weight is increased, without increasing the resistance of the air; and by this alteration its range is found to be increased also. The accuracy of fire is also found to be greater, as the angle of departure is diminished. Hence, greater accuracy of fire may be expected from heavy guns than from light ones, the sides of the shells in the former being much longer than in the latter, while the windage in both is equal. The following construction of a shell for a 9 pounder, I consider as the best, so far as the experiments made will allow me to determine. In these proportions, the length of the sides is increased half an inch beyond that of any yet used, and the twist is reduced from 55 to 72 inches.

Length	6,24 inches.
Diameter	4,16
Length of sides	4,16
Height of cone	2,08
Depth of grooves	0,2
Width of ditto round the circumference	0,8
Length of peg	1,4
Diameter of ditto	0,4
Diameter of vent	0,15
Thickness of sides	0,85
Thickness of bottom	0,74
Diameter of chamber	2,46
Height of ditto	3,42
Windage	0,04

In all the experiments already made, it has been observed that the line of fire is generally good, but that the shells which have missed the object, went almost invariably either too high or too low, which is exactly the result we might from theory expect, when their ends are not perfectly balanced. The method of proving whether their ends be exactly balanced, is by floating them in mercury, their specific gravity when filled being something less than half that of mercury. When properly balanced, they float horizontally, and the balance is not perfect until that is effected.

It is conceived that shells of this description might be used against towns and stockades, for battering in breach, and also in the field; but it is evidently in naval warfare that they would be most efficient, for the burning of shipping. That for a 24 pounder will weigh 30lbs. and contain $2\frac{1}{4}$ lbs. of powder, or what is reckoned better, powder and combustible

matter in equal parts. The composition of portfire is recommended, as it burns with great intensity, and is not easily extinguished. It may be ground to powder, and mixed with the bursting charge. If a shell of this size exploded on board a ship of war, it would be difficult to extinguish it under any circumstances, and if it passed through near the water-mark, almost impossible. Under these circumstances, ships lying low in the water, might possibly have an advantage over larger ones, from being less exposed to the fire of an enemy.

If ever the weapon should be used in war, it is only to be hoped that it may have a fair and impartial trial; and if it stand the test, the consequences may be considerable.

II. *On the relative powers of various metallic substances as conductors of electricity. By Mr.* WILLIAM SNOW HARRIS, *of Plymouth, Surgeon. Communicated by* J. KNOWLES, *Esq. F.R.S. November* 14, 1826.

Read December 14, 1826.

THE relation between metallic bodies, as conductors of electricity, has engaged the attention of those whose talents have, at various periods, enriched that branch of science; I enter therefore upon a further investigation of this interesting subject with much diffidence; but having, by an easy method, obtained a series of results, apparently calculated to advance our knowledge of it, I am led to hope that a short account of my inquiries may be honoured by the notice of the Royal Society.

It has been long since observed by one of the most active contributors * to the success of modern science, that the heat evolved by a metallic body, whilst transmitting an electrical charge, is in some inverse ratio to its conducting power—a principle generally admitted, not only as a reasonable deduction, but also as being established by a great variety of facts; I have therefore sought to measure the relative degree of heat, so evolved, by various metallic substances in a gazeous medium such as air, and thus to discover their precise relations as conductors of electricity.

I employed for this purpose a very simple instrument, (represented by fig. 1. in the annexed plate,) which may be

* Mr. CHILDREN's Experiments with a large Galvanic Battery.

considered as little more than an air thermometer, the metal to be examined being drawn into a wire and passed air tight through the bulb. There is a glass tube, whose interior diameter is regular, and somewhat less than $\frac{1}{10}$th of an inch; one of its extremities is bent upwards and outwards for about two inches, and is united by welding to a short glass cup; this last contains a small quantity of coloured spirit. The opposite leg of the tube is sustained by a graduated scale, fixed upon a convenient base. The point at which the coloured spirit rests in this leg is marked zero. Upon the cup is screwed a glass ball of 3 inches diameter, having the metallic wire to be examined passed air tight across its centre. To effect this, there are two flanches of brass carefully cemented about the holes in its sides, each flanch has a small projecting shoulder, which receives the wire, and upon which is finally screwed a small brass ball; this ball has a flattened part to bear against a similar part of the flanch, and thus by a thin collar of leather, the whole is rendered air tight. The wire is secured firmly in its situation by means of a small peg of wood, and the holes in the balls are sufficiently deep to allow both the extremity of the wire and the peg to project a little, for the convenience of removal; thus the substitution of one wire for another is very simple and expeditious. Besides these flanches and balls, the bulb is also furnished with a sort of valve, attached in a similar way to its upper part, which being rendered air tight by a screw, can be occasionally opened so as to form a communication with the external air, and thus the coloured spirit may at all times be adjusted to zero. Fig. 2. Under these circumstances when an electrical explosion of sufficient force is passed through

the wire in the bulb, the relative degree of heat it evolves is made evident by the ascent of the fluid along the graduated scale.

I submitted to experiment, by means of this instrument, the following metallic substances; silver, copper, gold, zinc, platinum, iron, tin, lead, alloys of gold and copper, of silver and copper, of silver and gold, brass, alloys of tin and lead, of tin and zinc, and an alloy of tin and copper. The above-named metals were carefully drawn through successive holes in a plate of steel until their diameters were the same, and in order to insure the transmission of an equal and similar explosion through each metal, I adopted the following method; two brass balls of the same dimensions were fixed at a given distance from each other, as in LANE's well known discharging electrometer; one of these balls, being insulated, was placed in immediate connection with the positive side of the battery, whilst the other was connected with the negative side, the metal to be examined forming part of the circuit. This last connection was effected by means of two fixed copper wires, inserted into the balls on each side of the glass bulb, and made perfect at the points of junction. When therefore the charge, accumulating in the battery, acquired a sufficient intensity to pass the given interval, the discharge took place through the wire in the bulb, which thus became immediately the subject of experiment.

The battery consisted of five jars, each containing five square feet of coated surface. They were placed on a metallic base communicating with the negative conductor, and were charged by means of long copper rods projecting immediately from the bottom of each jar.

The electrical machine employed to charge this battery consisted of a circular plate of glass, three feet in diameter, mounted between two horizontal supports of mahogany, the rubbers being insulated on glass pillars at each side of the plate, and joined together behind it by means of a curvilinear tube of brass, which formed the *negative* conductor; whilst the *prime* conductor projected vertically from the front of the frame.

The following table exhibits the results deduced from an extensive series of experiments on the different metallic substances above-named, in which the effect of the explosion is placed opposite the corresponding metal.

	Metals.	Effects.		Metals.	Effects.
	Copper	6	Alloys.	Copper 1 part, silver 1 part	6
	Silver	6		Copper 1 part, silver 3 parts	6
	Gold	9		Copper 3 parts, silver 1 part	6
	Zinc	18		Gold 1 part, silver 1 part	20
	Platinum	30		Gold 1 part, silver 3 parts	15
	Iron	30		Gold 3 parts, silver 1 part	25
	Tin	36		Tin 1 part, lead 1 part	54
	Lead	72		Tin 3 parts, lead 1 part	45
	Brass	18		Tin 1 part, lead 3 parts	63
Alloys.	Gold 1 part, copper 1 part	20		Tin 1 part, zinc 1 part	27
	Gold 3 parts, copper 1 part	25		Tin 3 parts, zinc 1 part	32
	Gold 1 part, copper 3 parts	15		Copper 8 parts, tin 1 part	18

There are some interesting circumstances observable by reference to this table. If we consider the heat to be in the inverse ratio to the conducting power, it appears; 1st. That the heats evolved from silver and copper are alike, as also those of iron and platinum, and likewise zinc and brass; whilst the heats evolved from lead and tin, compared with each other are as 2 : 1; the same may be said of zinc and gold, or brass and gold.

2ndly. Considering silver and copper as the *best* conductors, (being the least heated by the explosion) then the conducting power of

Gold to copper or silver is as -	2 : 3
Zinc or brass to copper or silver -	1 : 3
Platinum or iron to copper or silver -	1 : 5
Tin to copper or silver - -	1 : 6
Lead to copper or silver - -	1 : 12

3rdly. It may be observed that the conducting power of metals, when alloyed, is variously affected: thus the conducting power of gold and copper, or gold and silver, when alloyed together, is *less* than either of these metals in a separate state; and the difference in the conducting power increases with the quantity of the inferior conductor alloyed. Thus, gold one part, with copper three parts, had its temperature raised to 15° of the scale; gold and copper in equal parts to 20°; gold three parts, with copper one part, to 25°: the same may be said of gold and silver; whilst an alloy of copper and silver, in similar proportions, does not vary from either of the metals separately.

Tin and lead, alloyed, appears to give a conducting power formed by that of each metal taken singly, and varying, as above, with the quantity of the inferior conductor; thus, an alloy of lead and tin in equal parts, gives an effect equal to one-half the effect on tin, added to one-half the effect on lead, and so on; the same may be said of zinc and tin.

4thly. It is observable that a very small quantity of alloy may influence materially the conducting power; thus copper alloyed with only one-eighth part of its weight of tin, becomes

heated by an electrical explosion as much as iron. In accordance with this fact, it was found that wires drawn from some foreign gold coins, said to be very pure, were much worse conductors of electricity, than when drawn from the same previously refined.

I did not find the conducting power to be influenced by any new disposition or arrangement of the quantity of metal; thus, whether the metallic wire was perfectly cylindrical, flattened into a ribbon, or separated into four smaller wires, the effect produced was in each case alike.

The influence of a small portion of alloy on the conducting power renders it necessary to have the metals pure, and I have reason to believe that the specimens, which were in these instances made the subjects of experiment, were as nearly so as possible.

The alloys were prepared by fusing the metals together with a common blow-pipe on a charcoal support, having previously weighed the relative proportions; after which the small button of metal was again weighed and drawn into wire. I am not aware that this method of forming alloys with small quantities of metal is liable to any material error.

The wires operated on in the course of this investigation varied from the $\frac{1}{40}$th to the $\frac{1}{80}$th of an inch in diameter, below which it was not found desirable to reduce them.

EXPLANATION OF PLATE II.

abcd The glass bulb.

ab The wire.

d The brass caps and screws which unite the ball to the glass cup.

e The glass cup.

c A small valve.

ab The flanches and balls.

gfe The glass tube.

gf The scale.

s The stand.

P*a*. *b*N The connecting wires.

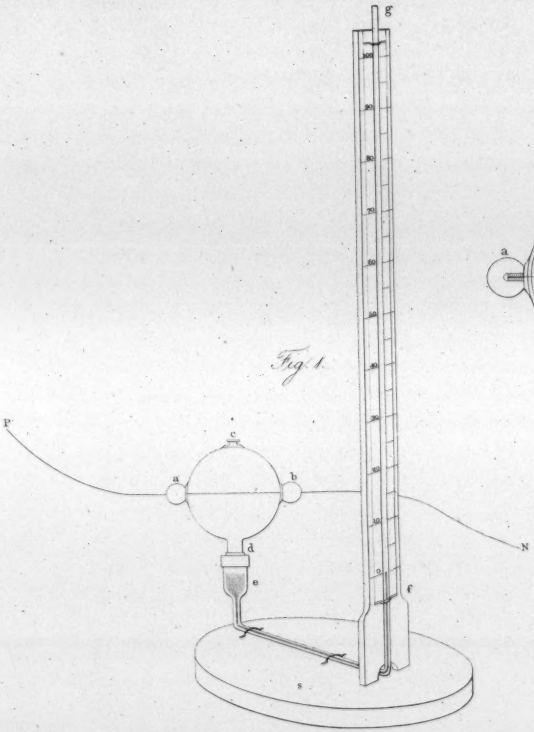


Fig. 1.

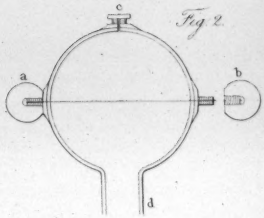


Fig. 2.

III. *On the expediency of assigning specific names to all such functions of simple elements as represent definite physical properties; with the suggestion of a new term in mechanics; illustrated by an investigation of the machine moved by recoil, and also by some observations on the Steam Engine.* By DAVIES GILBERT, *Esq. M. P. V. P. R. S. &c.*

Read January 25, 1827.

THE expediency of distinguishing by separate appellations, all such functions of simple elements as measure the intensity of physical properties, will be rendered obvious by referring to the well known controversy respecting motion.

Scarcely had the principles which regulate the action of bodies in motion become subjected to mathematical calculation, when a dispute arose as to the measure of motion itself; a dispute conducted with much more vehemence and acrimony than might be supposed incident to the nature of an abstract subject.

Several individuals of the greatest learning and reputation contended that weight, multiplied by velocity, ($w \times v$) gave a product always proportionate to the motion of bodies, as was proved by a comparison of their inertiæ, by all the properties relative to the common centre of gravity in planetary systems, &c. &c. &c.; while other persons, scarcely inferior to the former, adverting to the collision of elastic bodies, and to the extremely curious property of motion, the conservatio virium vivarum, contended with equal confidence, that the true measure of motion was weight multiplied into

the velocity squared ($w \times v^2$); till, in the early part of the last century it was fortunately observed, that the different properties indicated by these two functions were not, in any respect, at variance with each other; and the terms *momentum* and *impetus* reconciled all opinions, by removing every ground for dispute.

For a full and detailed explanation of this subject, I would refer to the admirable treatise on "The Rectilinear Motion and Rotation of Bodies," by the late GEORGE ATWOOD, Fellow of Trinity College, Cambridge, F. R. S. printed in 1784; and to the Bakerian Lecture in the Philosophical Transactions for 1806, by an existing member of this Society, who has never touched any point of science that he has not elucidated and adorned.

In this Lecture it is observed, that neither impetus nor momentum have usually much to do with the action of ordinary machines; which is undoubtedly true; since neither of these functions measures directly their efficient power. The criterion of their efficiency is force multiplied by the space through which it acts ($f \times s$); and the effect which they produce, measured in the same way, has been denominated *duty*, a term first introduced by Mr. WATT, in ascertaining the comparative merit of steam engines, when he assumed one pound raised one foot high, for what has since been called, in other countries, the dynamic unit.* And by this criterion, one bushel of coal, weighing 84lbs. has been found to perform a duty of thirty, forty, and even fifty millions; augmenting

* The dynamic unit used in France is a cubic metre (3,2809 feet) of water raised through a metre in height = 7220 pounds of water one foot high, being to a million as 1 : 138,5 as 72 : 10000 nearly; the log. 2,1413966.

with improvements chiefly in the fire place, which produce a more rapid combustion, with consequently increased temperature, and a more complete absorption of the generated heat; in addition to expansive working, and to the use of steam raised considerably above atmospheric pressure.

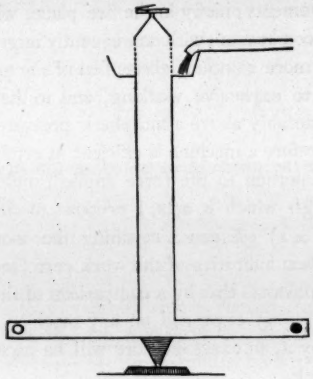
Since therefore a machine is efficient in producing duty, or effect, in proportion to the force applied, multiplied into the space through which it acts, I propose to denominate this function ($f \times s$) *efficiency;* retaining the word *duty* for a similar function indicative of the work performed.

And it is obvious, that by a comparison of these two quantities, the *efficiency* expended on any machine, and the *duty* performed by it, an exact measure will be ascertained of its intrinsic worth.

Having never seen a full investigation of the engine moved by recoil, I am partly, on that account, induced to take it as an example of the utility to be derived from the use of this new term; and I am in part also induced to do so, by the circumstance of a most respectable society having last year held out the properties of BARKER's mill to the public, as fit objects for inquiry.

To investigate the Recoil engine.

Let every thing be taken in the abstract, and after a manner most advantageous to the machine. Suppose friction to vanish, and that the perpendicular tube, and the arms, are so capacious as to allow of the fluid issuing with a velocity equal to what a body would acquire by falling in free space, through an elevation equal to that of the perpendicular tube, or through, what is usually termed, the head. See Principia, B. 2. § 7.



Let l = the space in English feet through which a body freely descends in one second of time.

Then $2l$ = the velocity acquired.

h = the length of the perpendicular tube, or the actual height of the head.

Then $2\sqrt{hl}$ = the velocity due to the head.

Let R = the length of either arm from the centre to the orifice.

x = the velocity with which that point moves in revolving, expressed by feet in a second.

Then $\frac{x^2}{2lR}$ = the centrifugal force at the orifice.

And since the force exerted by each particle of the fluid varies as its distance from the centre $\frac{x^2}{2lR} \times \frac{R}{2} = \frac{x^2}{4l}$ = the whole pressure from centrifugal force. An expression independent of the length of the arm; and the slightest reflec-

tion will make it appear that no alteration can arise from deflecting the form of the arm out of a straight line.

Then, since the pressure exerted by the actual head is expressed by its height (h), and the pressure from centrifugal force by $\frac{x^2}{4l}$,

$h + \frac{x^2}{4l}$ = the pressure sustained, or the virtual head, and $\left(h + \frac{x^2}{4l}\right) \times x$ = the efficiency applied to the machine, in relation however, not to the expenditure of fluid corresponding to the height (h), but to an expenditure greater in the proportion of $\sqrt{\left(1 + \frac{x^2}{4hl}\right)}$ to 1. But a portion of this efficiency must, of necessity, be expended in giving the rotary velocity (x) to the quantity of fluid issuing $= \sqrt{1 + \frac{x^2}{4lh}}$. Now the height due to x is $\frac{x^2}{4l}$, consequently $\sqrt{1 + \frac{x^2}{4lh}} \times \frac{x^2}{4l} \times x$ = the expenditure of efficiency in producing the rotary motion. And $\left(h + \frac{x^2}{4l} - \sqrt{1 + \frac{x^2}{4hl}} \times \frac{x^2}{4l}\right) \times x$ = the efficiency applied to the machine capable of producing duty.

Now substitute for x, (the velocity due to h) ($2\sqrt{hl}$) multiplied by an arbitrary quantity y; then $\sqrt{1 + \frac{x^2}{4hl}}$ (or the quantity of fluid issuing) $= \sqrt{1 + y^2}$ and $(y + y^3 - \sqrt{1 + y^2} \times y^3) \times 2h^{\frac{3}{2}} l^{\frac{1}{2}}$ = the efficiency; and dividing by $\sqrt{1 + y^2}$ $\sqrt{1 + y^2} \times y^3 \times 2h^{\frac{3}{2}} l^{\frac{1}{2}}$ = the efficiency that would be applied if no more fluid issued than what is due to the actual head (h).

The following Table exhibits the value of both these functions, taking (y) from o (where the duty must obviously be nothing) by steps of $\frac{1}{100}$ths to 1.25; after which the duty becomes negative.

y	The Duty for a given expenditure of fluid, on the function $\sqrt{1+y^2}\times y-y^2$	y	The Duty performed by the actual expenditure of fluid, or the function $y+y^2-\sqrt{1+y^2}\times y^2$
.05	.04994	.05	.05000
.10	.09950	.10	.10000
.15	.14830	.15	.14997
.20	.19596	.20	.19984
.25	.24207	.25	.24952
.30	.28621	.30	.29881
.35	.32794	.35	.34762
.40	.36681	.40	.38607
.45	.40234	.45	.41159
.50	.43402	.50	.48525
.55	.46132	.55	.52649
.60	.48371	.60	.56410
.65	.50062	.65	.59706
.70	.51146	.70	.62432
.75	.51563	.75	.64454
.80	.51250	.80	.65632
.85	.50145	.85	.65812
.90	.48183	.90	.64824
.95	.45297	.95	.62479
1.00	.41421	1.00	.58578
1.05	.36488	1.05	.52908
1.10	.30427	1.10	.45233
1.15	.23170	1.15	.35311
1.20	.14646	1.20	.22878
1.25	.04785	1.25	.07660
1.28	—.01803 negative	1.28	—.02929 negative

$l = 16.0954$ feet the log. 1.2067016

If the angular velocity of a body in one second of time be expressed in whole revolutions in degrees, in minutes, or in seconds, the absolute velocity will be =

$R \times 2cr = R'r \times 6.2831853$ . . . log. 0.7981799

$D^\circ \times \frac{2cr}{360} = D^\circ r \times 0.0174533$ . . log. 8.2418774

$M' \times \frac{2cr}{360\times 60} = M'r \times 0.00029089$. log. 6.4637261

$S' \times \frac{2cr}{360\times 60\times 60} = S''r \times 0.0000048817$ log.4.6855748

The first of these functions reaches the maximum, when y exceeds .75 or $\frac{3}{4}$th by two or three thousandths; and the second when y is about .83.

From an inspection of the Table, it appears that when the machine is moving with the velocity productive of its greatest relative effect, namely, at three quarter parts of the velocity with which the fluid would issue under a pressure equal to that of the actual head, that the duty exceeds by a mere trifle one half of the efficiency expended; and from thence it obviously follows, that the recoil engine cannot in any case be

employed with advantage. The water wheel, whenever the fall does not exceed forty or fifty feet; and the pressure engine, provided with an air vessel, in all cases where the fall is greater, receive an efficiency almost equal to the whole expended, instead of one half; and in practice, the duty will reach three quarters. The wheel moreover possesses a self-regulating power, sufficiently accurate for the present illustration, and somewhat analogous to the isochronism of the pendulum; that is, the actual weight of water on the wheel, and its velocity of rotation, must always be reciprocal to each other; thus maintaining a given efficiency, independent of velocity. The received efficiency of a water wheel being represented by the pounds of water passing over it x by the fall in feet, less the height due to the velocity with which the periphery moves in its rotation. But in the recoil engine, the moving power is expending at the rate due to the height even when the machine is actually standing still; and when it moves with a velocity exceeding that due to the length of the head, in the proportion of about 128 : 1 (where the duty is again nothing) the moving power will be expended at the increased rate of $\sqrt{1 + (1.28)^2} : 1$.

Here too may be easily shown the utter impossibility of executing a plan, which would not indeed deserve any notice, were it not that two ingenious practical engineers have within these few years, although at different times, and independently of each other, incurred large expense, and employed much pains in attempting to apply steam on the principle of recoil.

It is obvious that one of the factors, namely force (or the pressure of recoil), must in this case be extremely small, since an aperture of one inch square will discharge about $3\frac{3}{10}$ imperial

gallons of water each minute in the form of steam, unassisted by centrifugal force, while the pressure amounts to no more than 14,66 pounds. The other factor (velocity) must therefore be made proportionably great to produce an adequate effect.

Now, since air rushes into a vacuum with the velocity of 1295 feet in a second - - - log. 3.1122698

And steam at 212° is said to have a specific gravity compared to air of 0.6235

Log. 9.7948365 ÷ 2 = 9.8974183 arith. comp. 0.1025817

A velocity of 1640 feet in a second 3.2148515

Three quarters of this velocity in rotation will be requisite for imparting an efficiency to the machine of about one half of the efficiency expended. But such a velocity nearly equals that of a cannon shot, and the effect of centrifugal force on the arms $\left(\frac{x^2}{4t}\right)$ would produce a strain equal to the weight of 41776 feet of the material composing them, independently of the strain arising from centrifugal force of the fluid they contain. And as the modulus of tenacity for iron is estimated at 14800 feet, the centrifugal force will exceed that tenacity almost three times. But independently of this insuperable obstacle, any velocity adequate to the production of an useful efficiency must be utterly unmanageable in every other respect; nor is it easy to imagine the possibility of any expedient, similar to the air pump of Mr. WATT, for maintaining the vacuum in which their arms are to revolve.

The Steam Engine.

To estimate the efficiency of steam acting uniformly with its entire force, at a power corresponding to 30 inches of barometrical pressure, in reference to the consumption of one bushel of coal, I assume, from repeated experiments, that

14 cubic feet of water may be converted into steam, by that given consumption of fuel, occupying about 1330 times more space than the water from which it was produced. The whole space therefore filled by the steam amounts to 18620 cubic feet, capable of containing 1′150410 pounds of water; and as 30 inches of mercury at a specific gravity of 13.568 correspond to a column of water 33.92 feet in length, through this space the 1′150410 pounds of water may be supposed to be raised when a vacuum is formed, giving an efficiency of 39′361000, or above thirty nine millions of pounds raised one foot high.

But when this power of steam is applied through the medium of a piston, acting in the cylinder of a steam engine, large deductions must be made to arrive at the practical duty.

The air pump is usually constructed of a size to require about one eighth part of the efficiency;

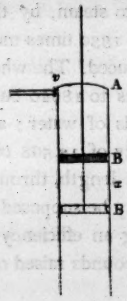
Some imperfection in the vacuum cannot be avoided;

And friction must exist to a considerable amount where bodies in motion have to confine an elastic vapour, ready to escape through the smallest aperture. So that thirty millions would probably have exceeded the attainable limit of duty, but for two expedients:

First. Causing the steam to act expansively, after exerting its whole force through a certain part of the cylinder; and

Secondly. Raising the temperature much above 212° of Fahrenheit, and thereby communicating to the steam a power beyond what is termed atmospheric strength; but this is, of course, accompanied by an additional expenditure of fuel.

The plan of working expansively is founded on the following investigation.



Let A B represent the portion of the cylinder through which the piston is urged by the entire force of the steam, and let its length be expressed by unity. Then let the supply of steam from the boiler be cut off by the valve at (v) and conceive the piston to have descended to B through the space (x) urged by the expansive elasticity of the steam, the density of which will then be $\frac{1}{1+x}$, and assuming that densities and elasticities are proportionate, $\frac{\dot{x}}{1+x}$ will be the fluxion of the efficiency, and the efficiency itself equal the nat. log. of $\overline{1+x}$. And the efficiency of the whole stroke from A to B will be 1 + nat. log. $\overline{1+x}$, giving this very curious result, that when x is infinite, the efficiency also exceeds any assignable limit.

$x=1$	$1+n.\overline{l.\,1+x}=1.693$
2	2.099
3	2.386
4	2.609
5	2.792
6	2.946
7	3.079
8	3.197
9	3.303
10	3.398

But it is obvious that several important circumstances have been omitted:

The reduction of temperature, and consequently of elasticity, caused by rarefaction, as the steam expands:

The increased relative effect occasioned by the reaction of the imperfect vacuum:

The increased relative burden of the air pump.

The relative increase of friction.

So that although this mode of working is highly advantageous, yet it is probable that, in practice, not more than from one half to two thirds of the cylinder can beneficially be occupied by expansive steam.

This mode of working has yet a further beneficial effect. The steam, beginning to act with its entire force, imparts the requisite velocity, by rapidly overcoming the inertia; and this force diminishing in the latter part of the stroke, allows the machine to exhaust the motion imparted, as it tends towards its close; thus also avoiding all violent percussion, which must obviously prove most injurious to the machinery, and in other respects be a pure loss.

It would be unfair towards the memory of a very ingenious man, were I to omit noticing, that much about the time when Mr. WATT introduced this method of using the expansive power of steam, a modification of the same principle was invented by Mr. JONATHAN HORNBLOWER, of Penryn in Cornwall. According to this method, instead of condensing the steam after it had passed under the piston, in the working cylinder, the steam is allowed to expand itself over a second piston in a second cylinder, when it is finally condensed in the usual manner; and if the second cylinder exceeds the first in the proportion of $1 + x$ to 1, the theoretical effect will be identical with that produced by the former method. But the mode invented by Mr. WATT excels the other in simplicity, and causes less friction.

The second expedient, that of employing steam at an high degree of temperature, and consequently of pressure, suggested itself in the following manner.

Experiments had shown, that a quantity of heat about nine hundred times greater than what is required to raise the temperature of a given weight of water one degree of Fahrenheit's thermometer, becomes latent in the conversion of that water into steam. That an elevation of temperature to

40° of Fahrenheit above 212° doubles the steam's elastic force, and that 30° more treble it; great expectations were consequently formed of the increased power that would be obtained, from a given consumption of fuel, by using high pressure steam; but the following most important circumstances were overlooked, till experience forced them into view.

That the temperature of steam cannot be raised, with a speed suited to mechanical purposes, in any other way than through the medium of its generating water.

And that water heated above 212° sends off more steam, with a proportionate absorption of latent heat, increasing the density of the steam previously generated of atmospheric elasticity, thus doubling its power at $212° + 40°$; but at an expense of fuel, not much inferior to the power gained.

If steam could be heated, as an independent elastic fluid, expanding $\frac{1}{480}$th of its bulk for each degree of Fah.; and if its capacity were the same as the capacity of water, a quantity of heat, equal to that rendered latent, would raise the power from unity to $\left(\frac{481}{480}\right)^{900} = 6\frac{1}{2}$. If the capacities of steam and water, according to some recent experiments, be as 1,55 to 1, the increase of power will be from unity to $\left(\frac{481}{480}\right)^{\frac{900}{1,55}} = 3\frac{1}{3}$. But the non-conducting property, common to all elastic fluids, has hitherto prevented this advantage from being obtained. Heated through the medium of its generating water, high pressure steam is believed however to be advantageous; and by its means additional power may be obtained, when an increase is requisite for any temporary use.

A contrivance has recently been attempted, by which a given small quantity of water is forced into a minute boiler at each stroke of the engine, and the boiler, presenting a very large surface in proportion to its content, is kept at an equable high temperature by immersion in a fuzed metal. Thus great degrees of elasticity and of rarity may be combined;

and the plan seems worthy of being fully prosecuted. But the main hope of augmented power, for still further elevating the condition of the human race, by increasing the empire of intellect over brute force, and for accelerating that splendid career, which, originating with the inventions of the plough and of the loom, has acquired such unhoped for rapidity in our own times, appears to rest on the application to mechanical purposes of some fluid more elastic than the vapour of water, according to the suggestion of our President in the Philosophical Transactions for 1823.

The following are a few instances selected from one of the Monthly Statements, published in Cornwall, of the work done by the principal steam engines, July 1826.

Mines.	Engine, and the diameter of the cylinder.	Load per sq. inch, on the piston.	Length of the stroke, in the cylinder	No. of Lifts	Depth.	Diameter of the Pump.	Time.	Consumption of coal, in bushels.	Number of strokes.	Length of the stroke, in the pump.	Load, in pounds.	Pounds lifted one foot high by consuming a bushel of coal.	No. of strokes, per minute.	Remarks, and Engineer's Names.
		Lbs.	Ft. In.		Fms. ft	Inches.				Ft. In.				
Wheal Vor	Trelawny's engine, 80 inches, single.	13,3	9 9	5 3 1 1 1	135 2 33 3 11 4 12 0 7 0	15 16 16 9½ 9	July 7th to July 31st.	2633	210490	7 0	89305	49,975,186	6,08	Drawing perpendicularly 135 fms.; and on the underlay 27 f. Main beam over the cylinder. Two balance bobs under ground. SIMS and RICHARDS.
Wheal Hope	60 inches single.	6,78	9 0	2 1 1 1	37 4 22 1 5 0 12 1	13 12½ 8 7	June 28th to July 29th	945	273730	8 0	21560	49,960,794	6,1	Drawing all the load perpendicularly. Main beam over the cylinder. 1 balance bob at the surface. GROSE.
Herland	Manor, 80 inches, single.	10,9	9 6	1 3 1	35 2 112 4 23 2	12 15 14⅞	June 27th to July 29th	2619	208880	7 6	72817	43,556,743	4,5	Drawing all the load perpendicularly. Main beam over the cylinder. One balance bob under ground, and one at the surface. WEBB.
Ditto...	Fancy, 80 inches, single.	7,6	9 6	2 1 1	72 0 30 3 21 0	14½ 14⅛ 13	Ditto	1825	213670	7 6	50890	44,686,299	4,6	Drawing all the load perpendicularly. Main beam over the cylinder. One balance bob under ground. WEBB.

From this document it appears, that several of the large engines now at work in Cornwall, are actually performing a duty about one quarter part greater than would be the whole efficiency of steam, unaided by expansive working, or by high pressure; assuming that fourteen cubic feet of water is the quantity converted into steam by a bushel of coal; but other engines, apparently similar in every respect, fail in performing half this duty; and no satisfactory cause has yet been assigned for the important difference.

As examples of the power and energies of the human mind, these applications of fire, through the medium of an elastic fluid, to mechanical purposes, stand pre-eminently distinguished: as elaborate contrivances useful to man, steam engines are confessedly without a rival.

No series of investigations can therefore be more worthy of minute and accurate attention than one which may enable us to connect by general laws, the temperatures, the densities, the elasticities, the capacities for heat, and the quantities rendered latent in assuming the elastic form; first for steam, and then, if possible, for all other bodies capable of existing in a gaseous state, both in contact with their generating fluids, and in complete separation from them.

I have gone more at length into the principles of the steam engine, than the immediate object of this Paper either requires or can warrant; but I trust that the Society will admit the great importance of the subject as an excuse; a subject with which local circumstances have rendered me familiar during the whole of my life.

IV. *The Croonian Lecture for* 1826. *By Sir* EVERARD HOME, *Bart. V. P. R. S.*

Read November 16, 1826.

THE subject of the present Lecture, is an enquiry into the mode by which the propagation of the species is carried on, in the common oyster, and in the large fresh-water muscle.

Aided by Mr. BAUER's microscopical observations, illustrated by his representations of the facts that were ascertained, I have been enabled to lay before the Society many curious particulars respecting self-impregnating animals, which could only be brought to light by an examination of the organs of generation in the field of the microscope; and without the continuance of his assistance, I confess myself unable to prosecute the enquiry.

It is now a period of five years since we entered upon the present investigation, continuing it during the breeding seasons of these two species of bivalves. Having at last brought our labours to a satisfactory conclusion, I shall now detail the observations we have made.

The singular fact of pearls having their origin in the abortive ova of these bivalves, has been already recorded in the Philosophical Transactions. Before that discovery was made, a pearl was imagined to be the nacral covering which the animal has a power of secreting upon any extraneous body, introduced by accident, or otherwise, between the shells, to render its surface equally smooth and polished with the shell

itself, and thus prevent it from injuring the substance of the animal in contact with it.

In the HUNTERIAN collection there are many specimens, in which extraneous bodies of different kinds have been introduced within the shells, through holes bored for that purpose, while the animal was alive; which in the course of time received an external coat of nacre, and bear a general resemblance to the pearl; some of these were glass beads, some leaden shot; but the lustre of the pearl cannot, I believe, be imitated, since it is produced by the bright internal surface of the central cell shining through the semitransparent coats which are afterwards formed upon it.

Although the ova of the oyster and fresh-water muscle agree in this one particular, of becoming the nucleus on which pearls are formed, the process gone through before the young is completely formed, is not the same in both species.

As the oyster is more simple in its structure, from having no organs fitted to give it the power of loco motion, which the muscle is provided with, I shall take its mode of propagation first into consideration.

In the whole range of comparative anatomy in which separate organs are developed for the three essential purposes of animal life, sensation, digestion, and propagation of the species, those organs in the oyster are the smallest, the most simple, and have the least to occupy them. Their mode of propagation will be found even more simple than it is in many plants, and the processes that are gone through, are carried on in a much shorter time.

As the following account does not, I believe, accord

entirely with those already before the public, either in this country, or in France, I have only to observe, that although others may have laboured the subject for a greater length of time, none, I am sure, have brought more diligence to the enquiry, or have more frequently revised the observations that were first made, with a view to correct any errors that were detected. With respect to the drawings by which the facts are illustrated, I can answer for their fidelity, should any voucher be necessary beyond the author's name.

On the mode of breeding in the Oyster.

The structure of the ovaria is so little developed, that it is difficult to discover these organs; and in the first instance, it requires the aid of the microscope for that purpose, even in the breeding season.

In this country, where the beds are not allowed to be disturbed during the season in which oysters spawn, we labour under a considerable disadvantage in the prosecution of this enquiry; and I am indebted to my friend Mr. Copeland Hutchinson, who procured for me the opportunity of examining some oysters weekly, during the period they are prohibited from being sold in the public market, taken from a private bed near Sheerness.

The situation of the two ovaria, for I consider them to be double, as in fishes, is immediately within the membranes that line the two shells, having the liver placed between them; they consist of a membrane, whose use is not to be ascertained till the ova become visible attached by pedicles, and hanging from it. The structure of the liver resembles so closely that of the ovarium, while containing ova, as only to be

distinguished by its more internal situation, and its colour, which is a shade darker.

In the month of March, the ova are so large as to be distinctly seen in the field of the microscope, and are then spherical; as they increase in size, the membrane to which they are attached becomes thickened. In June, they have arrived at their full size, and a white fluid like cream is now noticed surrounding them. That this is the impregnating liquor, is more than rendered probable by the ova dropping, or having dropped from their pedicles, and undergone a change in their appearance, a vesicle having formed, soon after which they leave the ovarium.

For this purpose a tube becomes visible, which before this period could not be detected; it originates by an opening between the two ovaria, which communicates with them both, and forms a sheath, in which the intestine is enclosed; it terminates externally by an orifice between the lips that surround the mouth of the œsophagus. This tube is the oviduct, which is single. The embryo is found when it enters the oviduct, to have already acquired a shell. The young begin to leave the ovaria in the end of June; and at the latter part of July none are to be found either in the ovaria or oviduct. On the 5th of August oysters are brought to market.

The ovaria, after the spawning has taken place, are not visible to the naked eye, but do not become evanescent, since in the microscope, fresh ova are seen in the very early part of their formation.

As fishes are what is called in high season while the ovaria are full of ova, it was to me not a little extraordinary that

oysters should, by common consent, be admitted neither to be good nor wholesome, under the same circumstances; for no general law against dredging for them would prevent those possessed of private beds from indulging their appetites during the months of May, June, and July, if they found that oysters were better in those months.

I had an opportunity last July of setting this question at rest; for being at Dieppe, in France, where there is no restriction laid upon oysters, I ordered some for dinner: they had no flavour, none of the company could eat them. I found it equally true in Paris; but the fact is, the period respecting the oyster, which corresponds to the breeding season of fishes, is March and April, when the ova are getting ready for impregnation. In June and July the ova have been impregnated, and may be said to have spawned at the time the embryo is first received into the oviduct, which is in the month of June. At the time the young oysters leave the oviduct, there is a mucus of a purple colour which is voided at the same time, probably for the purpose of supplying them with nourishment while they remain enclosed within the enveloping mantle by which the gills are surrounded.

While in this situation they often become a prey to small sea-worms, which get between the shells, and gorge themselves with the young ones. I have seen these worms with their stomach completely distended with young oysters.

There are many curious structures met with upon the edge of the mantle which encloses the gills. As these appear enlarged in the breeding seasons, their uses may be applicable to the growth of the animal, of the shell, or the formation of nacre. Accurate representations of them are given, that others

may be enabled to consider them, which is no part of the present enquiry.

The stomach and intestine of the oyster I have upon another occasion considered and delineated, and laid before the Society.

The heart has a near resemblance to that of the teredines, having two auricles and one ventricle.

On the mode of breeding of the fresh-water muscle.

In this bivalve, the ovaria in their situation and appearance are the same as in the oyster. The ova arrive apparently at the same size before they are impregnated, which in them also takes place in the ovaria.

The ova, while attached by their pedicles to the membrane of the ovarium, have an appearance only to be distinguished from the structure of the liver by the difference of colour.

About the 10th of August the ova are completely formed in the ovaria, and are detected about the 20th of the same month passing into the oviduct, which is a curiously trellised structure situated between the membranes that compose the bronchiæ; and about the 12th of September they have all arrived there.

That impregnation has preceded this change of situation is evident from the ovum having been formed into a vesicle, through the coats of which vesicle, very soon after it has been retained there, the embryo is distinctly seen surrounded by a fluid, opening and shutting the incipient shells for the aeration, and probably the nourishment of the fœtus in this stage of its growth.

While in this situation, many of the young were seen

turning round as it were upon a centre. This motion had been taken notice of by LEWENHOEK, who thought it so extraordinary that he did not wish the fact to rest upon his own evidence, and called his wife and daughter, that they might bear testimony of its having taken place. When Mr. BAUER first met with it, the same notion occurred to him, of wishing to have other witnesses than his own eyes. He called in a young female servant, and having directed her eye upon the object, he asked what she saw?—a little white thing turning round and round.

This revolving motion of the embryo very naturally attracted my particular consideration; and having seen the porcelain manufactory at Worcester, it bore so close a resemblance to the circular motion given to the pieces of clay out of which plates and saucers are formed, that for some time I was completely deceived; but Mr. BAUER's close and persevering examination very soon detected the true cause of this strange phenomenon, which was produced by a small worm that had got into the vesicle, and while feeding on the embryo, performed these revolutions, carrying the young muscle round along with it, although itself concealed from the eye of the observer.

The young remain in the oviduct, the interior of which has a greater resemblance to the honey comb in which the young bees are deposited, than any thing else I am acquainted with, till they arrive at the size fitting them to provide for themselves; they leave the oviduct in October and November.

When the young are ready to leave their cellular prison, a canal is formed through which they pass out; and as the

foot of the parent muscle is partly surrounded by a portion of the oviduct, when the foot is extended in the progressive motion of the animal, that portion of the oviduct is also carried beyond the external shells, so that the young will have every facility in being set entirely at liberty. This happens, as I have mentioned, in the months of October and November, towards the end of which they have all escaped; and even at this time young ova are found in the ovarium, preparing for the next season.

EXPLANATION OF THE PLATES.

Plate III. Fig. 1. An oyster with the convex shell uppermost, to identify the species to which it belongs.

Fig. 2. The same oyster, the convex shell being removed, showing the whole oyster, with the entire cloak, &c. &c.

Fig. 3. The same oyster, the cloak and one layer of the beard being removed, to show the ovarium and oviduct; from the edge of the ovarium is a small slice cut off, to exhibit the ova in their natural situation in an early state of pregnancy.

Fig. 4. An oyster in the act of emitting its living young ones, which are enveloped in a purple coloured mucus.

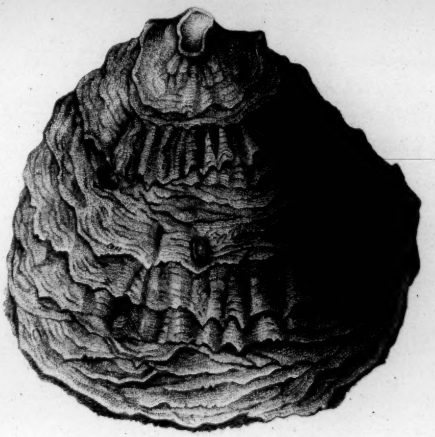
N. B. All these four figures are natural size.

Plate IV. Fig. 1. A perpendicular section of an oyster, to exhibit the course of the alimentary canal, the oviduct, the heart, &c. &c. the cloak and the beard being removed; magnified 2 diameters.

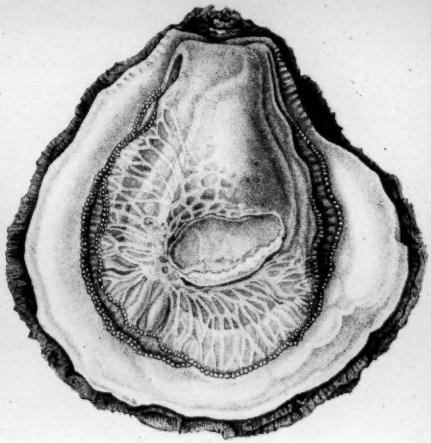
Fig. 2. A very small portion of the ovarium, with the ova imbedded in its substance; magnified 100 diameters.

Fig. 3. Some ova extracted from the same ovarium, as they

1.



2.

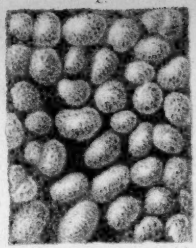


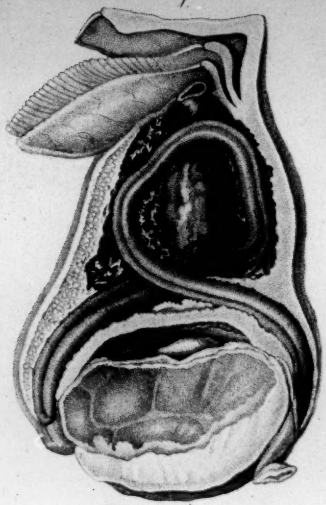
3.

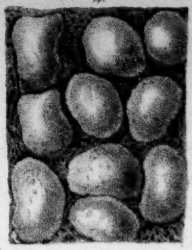
4.

Francis Bauer del.

Fd. Bauer sc.


2.


1.


4.

3.

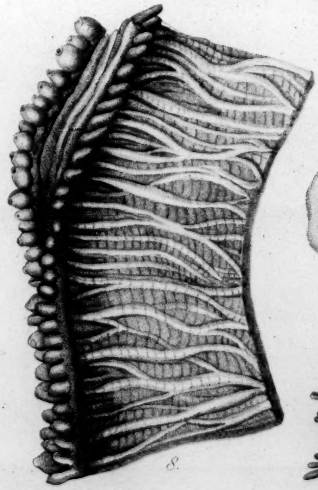
a.

5.

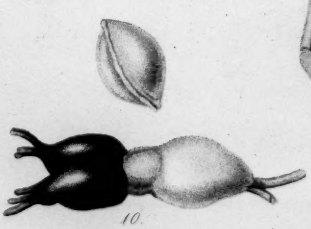
a

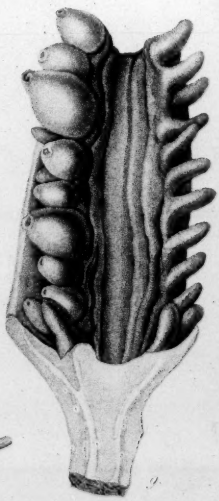
b

6.


8.

7.

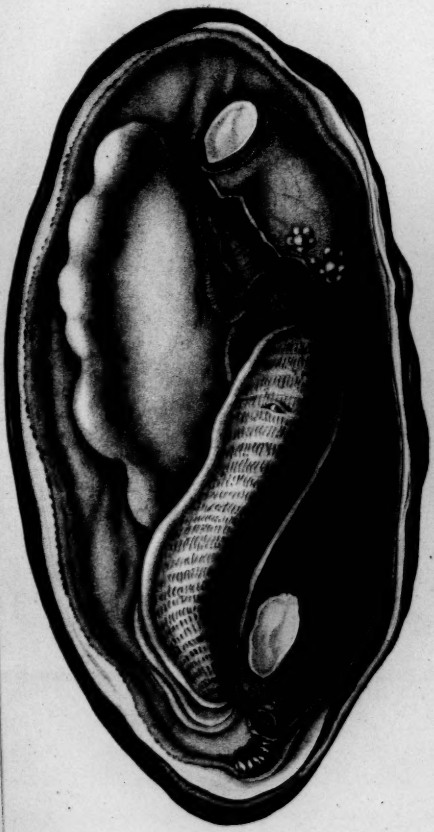

10.


9.

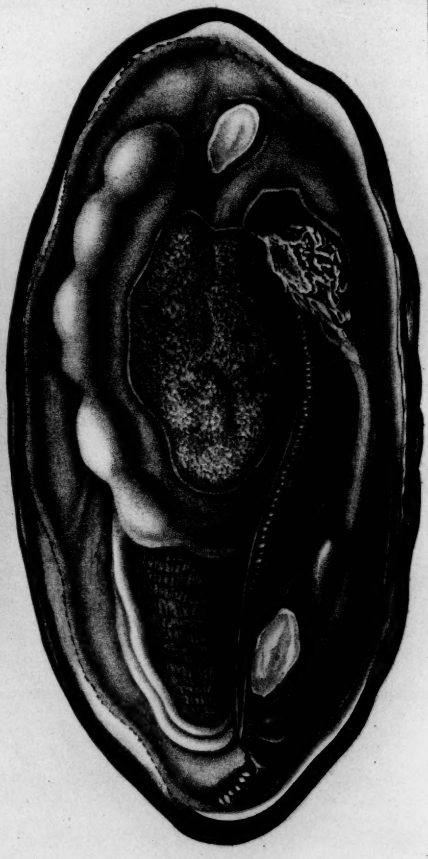
Franz Bauer. del.

Fr. Bauer. sc.

1.

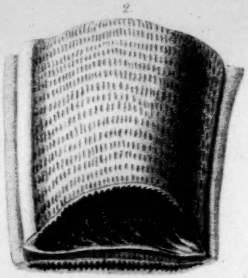


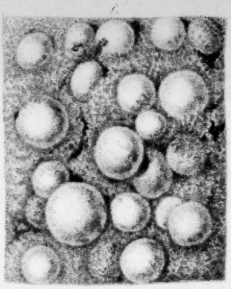
2.

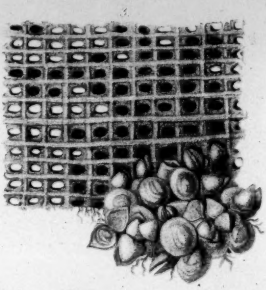


Fanny Steers del.

C. Rolls sc.

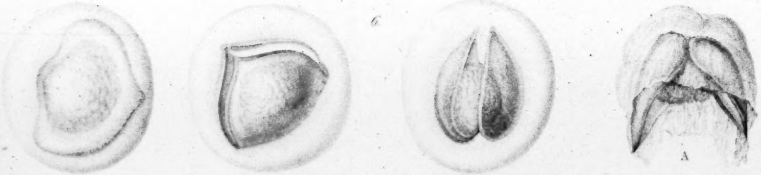

2.


1.


3.

4.

5.


6.
A

7.

8.

appear when floating in water, when they assume nearly a spherical form: at *a* is one of the ova after having been about one hour in water, when they dissolve into a granulated mass; magnified 100 diameters.

Fig. 4. An equally small portion of an ovarium in a more advanced state of pregnancy; magnified 100 diameters.

Fig. 5. Some ova extracted from the above portion of ovarium, floating in water, magnified 100 diameters: at *a* is one of these ova bursting, and emitting its granulated substance; magnified 100 diameters; at *b* is an empty, unimpregnated probably, ovum; magnified 100 diameters.

Fig. 6. Several young oysters just emitted from the oviduct, and floating in water and their natural mucus; magnified 100 diameters.

Fig. 7. Some of the same young oysters, after having been some time in contact with the air, when they become quite opaque and assume a glossy appearance; magnified 100 diameters.

Fig. 8. A small portion of the cloak with its fringe; magnified 25 diameters.

Fig. 9. A portion of the fringe spread open; magnified 50 diameters.

Fig. 10. The heart, with its auricles in the natural position, as it lies in the oyster; magnified 4 diameters.

Plate V. Fig. 1. A fresh-water muscle, one shell, and its lining or cloak removed; natural size.

Fig. 2. The same muscle with the oviduct removed, and a perpendicular section of the ovarium; natural size.

Plate VI. Fig. 1. A very small portion of the ovarium, with

the ova imbedded in the yellow granular substance of the ovarium, magnified 100 diameters.

Fig. 2. A transverse section of a portion of the oviduct; magnified 2 diameters.

Fig. 3. A small portion of the inside of the oviduct, with some of the ova ready for emission; magnified 20 diameters.

Fig. 4. Several ova, of various sizes, extracted from the ovarium; magnified 100 diameters.

Fig. 5. Ova, in the early state, extracted from the oviduct, and floating in water; magnified 100 diameters.

Fig. 6. Ova nearly ready for emission, extracted from the oviduct, and floating in water; at A is one just bursting its enclosing membrane or bladder; magnified 100 diameters.

Fig. 7. Some young muscles, just emitted, floating in water, where they are opening and closing in a very lively manner for several hours; magnified 100 diameters.

Fig. 8. Is the origin and progress of a singular worm, which is found within the oviduct of the muscle: originally it is an extremely minute globule, which is only to be distinguished from the usual granular substance by a curious rotatory motion, which it performs incessantly until it becomes quite organised; and it attains sometimes the length of an inch; magnified 100 diameters.

V. *On a newly discovered genus of Serpentiform Fishes.* By I. Harwood, *M. D. F. L. S. Professor of Natural History in the Royal Institution of Great Britain.* *Communicated by* Daniel Moore, *Esq. F. R. S.*

Read February 1, 1827.

In no department of natural history have descriptions been more unsatisfactory than such as relate to certain productions of the ocean, which, either from the immeasurable depths that conceal them, or the absence of those circumstances best adapted to their multiplication, very rarely present themselves to our notice, and from this rarity often excite impressions on our minds, ascribing to them properties foreign to their real natures, and at variance with that harmony which, even in the deepest recesses of the ocean, pervades the works of Omnipotence.

It is doubtless from a want of more frequent opportunities for investigation, that the ancients were induced to consider the sea as the abode of monsters and prodigies of the most incongruous characters; for, in addition to their prevailing opinion expressed in Pliny, "ut quidquid nascatur in parte naturæ ullâ, et in mare esse; præterque multa quæ nusquam alibi," we know that tritons, sirens, mermaids, and more lately krakens, and serpents of vast proportions, and varied properties, have been the frequent subjects of serious consideration; and even in the present day there are perhaps few whose imaginations, at some period of their lives, have not

been highly interested by the contemplation of such ideal beings. As the difficulty therefore of eradicating error from the mind, when once received, imparts value to truth, though least adorned, on this ground I would presume to present to the attention of the Royal Society an account of a newly discovered, and a very extraordinary marine animal. Last autumn, whilst Captain SAWYER, of the ship Harmony, of Hull, was in pursuit of the bottle-nosed porpoise, in latitude 62° north, by about 57° west, he observed a body floating on the surface of the water, which was at first mistaken by himself and his seamen for an inflated seal's skin, such as the Esquimaux employ in the destruction of large aquatic animals, by attaching it to the harpoon by which they are speared, and thus tiring them out by its floating property. On a nearer approach however, the object which had excited their attention proved to be a living marine animal. The creature is still in the possession of Captain SAWYER, who preserved it in rum soon after being taken, and who obligingly afforded me an opportunity of examining it. Its capture was occasioned by its being, when first observed, almost worn out by unavailing efforts to gorge a fish of about seven inches in circumference, with which it appeared to have been long contending, as it exhibited very feeble signs of life. Its organs of motion being extremely small, and its body greatly elongated, this creature would, on a cursory view, be by all considered as an extraordinary kind of sea serpent—a tribe of animals concerning which so much equivocal matter has been written; and this idea would be even supported by a more close examination of some parts of its structure. The fact, that the sea contained animals nearly allied in form to

serpents, has been remarked in the earliest ages; ARISTOTLE has expressly stated, that there are many species of sea serpents, among which tribe he evidently includes the branchial apodes, for he has particularized one of these as par excellence the Ὄφις θαλαττιος. This is the muræna serpens of LINNÆUS, or the ophisurus of LACEPÈDE, a creature of a more cylindrical form than the eel, and possessing other points of resemblance to the snake tribe; but although, in the present day, the term sea serpent would be ill applied to any animal which breathes by means of branchiæ, yet among such creatures, excluding the genera hydrus and hydrophis, and other true water snakes which inhabit tropical seas, I doubt if the subject of this communication be not at least as well entitled to that appellation as any hitherto described. From the several genera of animals however nearest allied to it, it offers points of disagreement so important, as to entitle it to a distinct place in classification, and especially from the formation of the jaws, which, with the exception of the apparent want of serpentiform inter-articular bones, are truly analogous to those of snakes; and, secondly, from the possession of an enormous elastic sac, which is seemingly a receptacle for air only. The first of these latter characters appearing to be the one, of all least liable to vary, I would suggest the term Ophiognathus as applicable to the genus; its characters are as follow:

OPHIOGNATHUS. Corpus nudum, lubricum, colubriforme, compressum, sacco amplo abdominali.

Caput anticè depressum, maxillâ superiore (paulo) longiore.

Dentes, in maxillâ inferiore, et ossibus intermaxillaribus, subulati, retroflexi.

Maxillæ elongatæ, patulæ, dilatabiles, (serpentium instar).

Lingua vix conspicua.

Spiracula ante et sub pinnas pectorales, magna.

Pinnæ pectorales, dorsales, analesque radiis mollibus; ventrales nullæ.

Oculi minimi, prope extremitatem maxillæ superioris positi.

Cauda elongata, in filamentum apterum producta.

As a genus, then, the association of the above characters distinguishes this creature from any others which I am aware have hitherto been described, although some of them may be met with in those genera nearest allied to it, as the muræna, ophisurus, gymnothorax, sphagebranchus, synbranchus, gymnotus, ophidium, gymnetrus, and trichiurus. In these however we find the following points of dissimilitude. In muræna the fins are adipose; ophisurus has teeth on the ossa palati, and gymnothorax has no pinnæ pectorales. In sphagebranchus the same fins are also absent, or imperfect, and the snout lengthened. In synbranchus the spiraculum is single; in gymnotus the dorsal fin is wanting. The genus ophidium has open branchiæ and large opercula; in gymnetrus the anal fin is deficient; and lastly, in addition to other important differences, this is supplied in the trichiurus by a dentated edge.

We now proceed to consider the only known species of Ophiognathus, which, from a character altogether anomalous among apodal fishes, we shall term *ampullaceus*.

Its body is one uniform purplish black, except the filamentous extremity of the tail, which is much lighter. The total length of the specimen taken is 4 feet 6 inches. The enlarged and extremely elastic pharynx communicates with the enor-

mous sac or air vessel, which extends in length from the extremity of the snout about 20 inches. The great delicacy of the parietes of this sac, and its apparent liability to rupture from the action of the spirit, prevented my inflating it to its full extent; but when partially filled with air, it measured about 9 inches in circumference below its union with the tail, and its greatest diameter, including the slender body to which it pertained, was 4 inches. At about one inch below the last point of its attachment with the body, the rectum was observed to perforate the sac, the tenuity of which rendered the course of that intestine, as indeed that of all the digestive organs readily traced. They are apparently sustained beneath the ribs (which latter appear very imperfect), by a membranous expansion, as they are not affected in their position by any inflation or emptying of the sac. That the sac itself communicates with the pharynx, is sufficiently proved by the fact, that if the blowpipe were further introduced, the digestive organs were alone inflated, the sac undergoing no change in its dimensions. The nearest point of analogy therefore to this structure observable in the class Pisces, is seen among the diodons and tetraodons, where a kind of ingluvies, or crop, formed of a very thin and extensible membrane, adheres closely to the peritonæum throughout the whole extent of the abdomen, by means of which their curious and rapid power of inflation has long excited surprise, though we cannot in the smooth Ophiognathus so satisfactorily account for its use, as among these fishes, which by its distention, mechanically elevate their spines, and thus float about in safety. This external sac might again be compared with the more internal air vesicle of most other kinds of fishes, and especially that

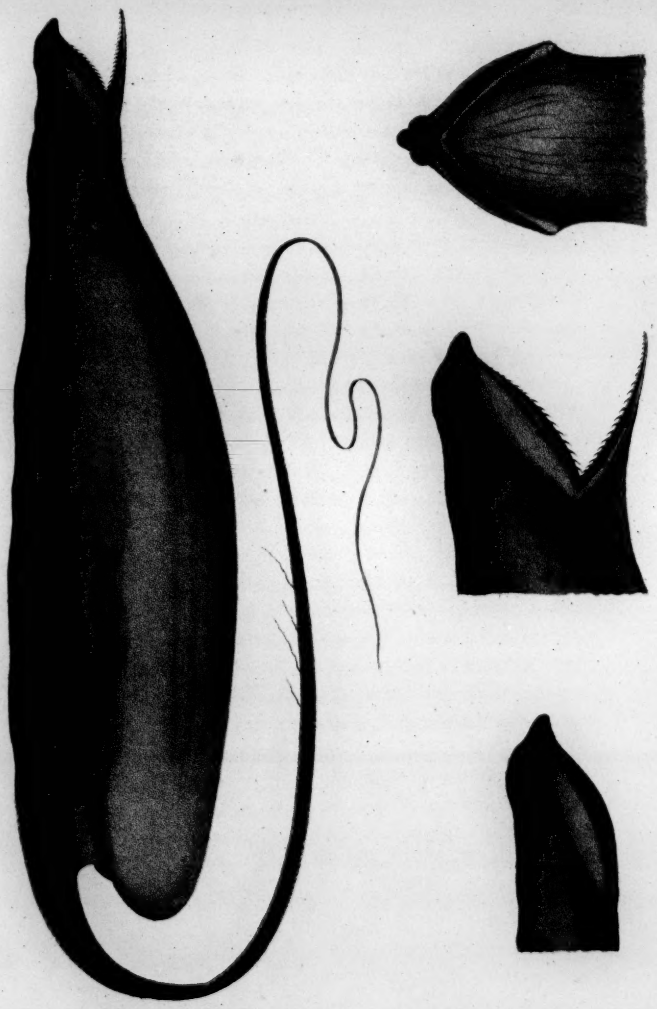
of the sturgeon kind, which has so large and free a communication with the œsophagus; but in extending our enquiries on this subject further, we find no class among the vertebral animals which does not offer some modification of a similar structure; for even among the mammalia, the whales, which are called the balænæ boops and rostrata, also the balæna musculus, if it be a distinct species, have the anterior part of the body covered with an extremely elastic skin, plaited into deep folds, as Mr. HUNTER has expressed it, "like unto a ribbed stocking," and capable of a vast extent of dilatation. This is partly exhibited in the act of swallowing food, and appears reasonably intended, by communicating with the external air, to render the body so specifically light, as to enable them to sleep with greater ease upon the surface. Of this communication however, that accurate observer, Mr. HUNTER, has taken no notice, but adds, "why the skin should be so elastic, is difficult to say, as it covers the thorax, which can never be increased in size; yet there must be some peculiar circumstance in the economy of the species requiring this structure, which we as yet know nothing of." Among birds, several examples occur of the possession of external air vessels of considerable size, which in like manner appear to be intended to more perfectly increase or diminish their specific gravity at their will. Of this structure the gigantic stork, ardea argala LIN. presents a striking example. Lastly, among reptiles, such mechanical aids, either internal or external, are far more numerous, for to this end we must consider the large internal vesicle forming an appendage to the lungs of turtles; and probably to the same end is the saccular and membranous inferior half of the lung in snakes, which is at

the same time transparent, and exhibits few traces of vascularity. To these might also be added many other instances; even among the lizards of the genus anolis, the lacertæ strumosa and bullaris, have beneath the anterior part of the body and throat, large elastic vesicles, which at will they quickly distend with air, though these species are not aquatic in their habits. But to return: the skin all over the body of the Ophiognathus is particularly soft and slimy, yet it has a slightly granulated appearance. The spiracula are of large size, of an irregular oval form, and are unprovided with externally perceptible branchiostegous rays; their edges partially conceal on each side three branchiæ. When the sac is contracted, these apertures are placed almost under the body, as in the sphagebranchus, having a narrow commissure between them; they are placed at about $5\frac{1}{2}$ inches from the snout.

All the fins of the Ophiognathus are extremely small; in the formation of the pectorals there is a peculiarity not mentioned, I believe, in other apodal genera, they being principally composed of an adipose disk, which is terminated, and nearly surrounded by a narrow radiated membrane, instead of this latter taking its origin immediately from the body. The dorsal fin, which like the rest is very narrow, and provided with simple rays, commences at about 18 inches from the snout, and terminates insensibly upon that slender, tape-like filament, into which the tail becomes converted, and which is continued $20\frac{1}{2}$ inches in length beyond the posterior extremity of the dorsal fin. About this part of the dorsal fin a few other minute filaments take their growth from it. The anal fin commences at the posterior union of the sac with the body, and ends at about 14 inches from the extremity of the

caudal filament. The body exhibits no apparent lateral line; but perhaps the most curious structures which the creature presents to our notice, are connected with the head and jaws. The almost entire absence of a tongue, might perhaps prove one of its most characteristic distinctions, were we as yet sufficiently acquainted with the condition of this organ in those nearest allied to it. The teeth are disposed in a single row above and below; above, they exist only along the margins of the intermaxillary bones; below, they extend almost the whole length of the maxilla; but the ossa palati are entirely destitute of teeth. Lastly, the jaw-bones are so long, and their articulation is such, that their capability of expansion exceeds what I have seen in any other animal, the rattlesnake not excepted; and as in snakes, when fully distended, the edges of the jaws describe a large circle, and then appear but as the hemming of an ample sack, the pharynx, which usually occupies so small a space, being an equal participant in this extensile property. When the jaws were gently opened, they measured $2\frac{1}{2}$ inches across, and $3\frac{1}{2}$ inches from the front teeth above to those below; but while they possess this capability of distension, their contractile power is no less remarkable, as may be observed in Plate VII. fig. 1. which represents the usual appearance. Fig. 2. exhibits the jaws and pharynx more depressed and extended. Fig. 3. represents an anterior view of the same.

Thus provided then, we find the Ophiognathus to be one of the most predatory and voracious of all the belligerent inhabitants of the ocean, as was proved by the efforts of Captain SAWYER's specimen to gorge a species of perch of more than its own usual circumference, in striving with which, as before



J. Harwood. del.

J.s Basire. sc.

observed, it appears to have lost all its muscular energy. Its entire form would indicate great swiftness of motion, which is doubtless effected by means of the same interesting sinuous inflections in the water, which excite our admiration in the class of serpents upon land; as it is indeed well ascertained, that some serpentiform fishes, especially the true murænæ, are capable of transferring with great effect, their aquatic locomotive powers to the surface of the earth. In what manner its enormous pouch is employed in its economy, I repeat, that I cannot presume to determine; its complete inflation with air, when on the surface, must, it would appear, afford a very effectual obstacle to the descent of the animal beneath. Whether it be capable of secreting the contained air, as has been thought of the common internal air vesicles of some other fishes, for which apparent purpose, eels have a peculiar gland connected with its centre, or whether water be allowed to enter its cavity, as it has been thought to enter the abdominal cavity of some rays, would form a subject of interesting enquiry. Having however entered on the field of conjecture, I shall not presume to longer engage the attention of the Royal Society, though I cannot but express a hope, that the discovery of another curious link in the vast chain of being, and especially in latitudes where such an one might have been least expected, will be considered sufficiently important to merit their notice.

Royal Institution, Jan. 10, 1827.

VI. *An examination into the structure of the cells of the human lungs; with a view to ascertain the office they perform in respiration. By Sir* EVERARD HOME, *Bart. V. P. R. S. Illustrated by microscopical drawings from the pencil of* F. BAUER, *Esq. F. R. S.*

Read February 8, 1827.

No subject connected with physiological enquiry has more excited the attention of the anatomist and chemist, than respiration; but the association between this subject and animal heat, which has so long been supposed to exist, has led to the belief, for the last century, that both enquiries belong more particularly to chemistry than anatomy, and I may probably be considered as going out of my province in taking up this investigation. On the other hand, I see reason to believe that the process of respiration is in itself more simple than is imagined, and more within the reach of discovery by means of accurate anatomical knowledge of the parts employed, than by means of acquaintance with the intricacies belonging to chemical affinities: I carry this so far as to contend that no explanation of respiration upon chemical principles is to be depended on, unless it accord in all respects with the anatomy and physiology of *the lungs*, by which the assumed process takes place.

The present theory respecting respiration adopted by the chemists, is, that this process decarbonises the blood in the following manner; at every inspiration a compound of

oxygen and nitrogen, mixed together, is received into the lungs, and in every expiration, the same volume is returned, measure for measure exactly, with this only difference, that what entered as oxygen is returned in the form of carbonic acid gas, which, according to *their* theory, proves that no part of the inspired atmospheric air has been retained in the lungs, but a quantity of carbon, equal to that of the oxygen inspired, has been extracted from the blood by the oxygen, making it become carbonic acid gas.

Nothing could be more ingenious than this theory, were it supported by the structure of the lungs themselves, and it could be proved that the blood requires no other changes for its purification; since all the leading facts on which it is founded, are completely established upon the firm basis of experiment.

When this theory was formed, the structure of the air cells of the lungs had never been examined, the more minute structures in animal bodies being at that time considered beyond the reach of examination; and it is the object of the present communication, to bring forward an explanation of the mechanism of the cells of the lungs, as well as of the different distributions of the vessels that ramify through those organs, acquired from Mr. BAUER'S microscopical observations, and to see how far they are fitted for the process, which by this theory is allotted to them.

In this investigation, I began by an enquiry into the circulation of the blood through the lungs, in the labour of which I have been very ably assisted by Mr. RUSSELL, a very intelligent student of St. George's Hospital, at present filling the highly respectable and important office of senior House

Surgeon. To him I am indebted for having taken the trouble of making injections of the arteries, the veins, and of the cells of the lungs, with different substances, so as to enable Mr. BAUER to expose and examine them on the field of the microscope.

The first new fact discovered in the course of this enquiry was, that although the common minute injection used by anatomists for filling the blood vessels, when thrown in by the trunk of the pulmonary artery, while the cells of the lungs are empty, returns again by the trunks of the pulmonary veins, yet when thrown in by the veins, it is not returned by the trunks of the arteries.

Another fact was discovered; that during the momentary distention of the air cells, an interruption is produced between the arterial and venal circulation in the lungs, the blood being carried no farther than the small arterial branches surrounding the air cells.

The following description of the air cells, and the parts surrounding them, is taken from the annexed microscopical drawings of Mr. BAUER.

As accurate representation surpasses all verbal description, I shall not have occasion to do more, to make myself understood, than to mention the parts themselves, and the circumstances under which they are represented.

The cells of the lungs were filled with quicksilver, to show their utmost capacity, and the parts were afterwards immersed in rectified spirit, to prevent the cells from collapsing, when the quicksilver was allowed to escape.

When the internal cavity of a single cell was exposed, immediately behind its internal membrane, the branches of

the pulmonary artery, injected with red wax, were seen ramifying, as arteries do in common; these were accompanied by branches of the pulmonary vein, larger in proportion than those of the arteries, more numerous, and having valves, at apparently regular intervals, to prevent regurgitation of their contents. Besides the arteries and veins, there were innumerable absorbents opening into the cavity of the cell; their valves were at very short distances, and, in their course in the interstitial substance between the cells, they accompanied the veins. When the terminal branches of the pulmonary artery were traced, the injection was found to have stopped someway before the artery's termination, and the space beyond was filled with gas. The substance of the lungs, interstitial to the cells, when dried became transparent, and was found to be composed of a smaller order of cells, with transparent coverings, that freely communicated with one another, as well as with the cavity of the large cell they surrounded.

I cannot finish this description of an internal view of one of the cells of the human lungs, without expressing the obligations I am under to Mr. DOLLOND, who from a zeal for science, and the peculiar interest he has taken in these physiological enquiries, has fitted up the microscope (from the field of which the representations were taken), possessed of advantages beyond all the others that Mr. BAUER has met with, not excepting the achromatic microscope, as improved by Mr. CHEVALIER of Paris. This of Mr. DOLLOND, from the superiority in the clearness of the different glasses, and the facilities which are given to the variations of the nicer adjustments of the instrument, has enabled Mr. BAUER to give a

more distinctly defined outline to all the parts, than he thought it possible to produce, when objects are so highly magnified.

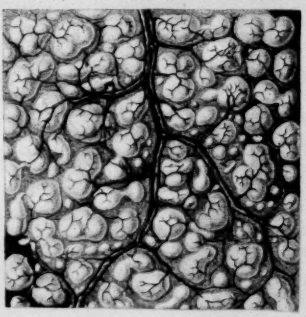
From this account of the anatomical structure of the lungs, it is evident they are calculated not only to receive supplies from the atmosphere, but to convey a part of them, with the greatest rapidity, as well as facility, to the heart; since the momentary interruption to the passage of the blood from the arteries to the veins, and the numerous valves in the absorbents, as well as those in the veins, are admirably fitted for that purpose; which is at variance with the theory of decarbonising the blood.

As carbonic acid gas has been occasionally detected by Professor BRANDE, both in the urine and perspirable matter, it must have been formed in the blood circulating through the arteries; and the supply of oxygen which enables this to take place, is now shown to be derived from the lungs.

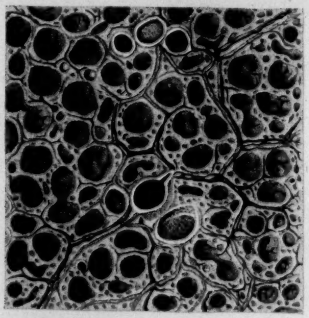
This mode of decarbonising the arterial blood, by a portion of its oxygen uniting with the carbon, and carrying it off in the form of carbonic acid gas by the urine and perspirable matter, is extremely simple, and appears to require no elaborate chemical process.

Having ascertained upon a former occasion, that soon after digestion has been begun, the oxygen employed in that process unites with carbon, and the quantity of carbonic acid gas, met with in the venous blood, is greater than at any other time; this will sufficiently account for the blood in the branches of the pulmonary artery always containing a sufficient quantity of this gas to replace the oxygen that is removed from the cells of the lungs, and carried to the heart;

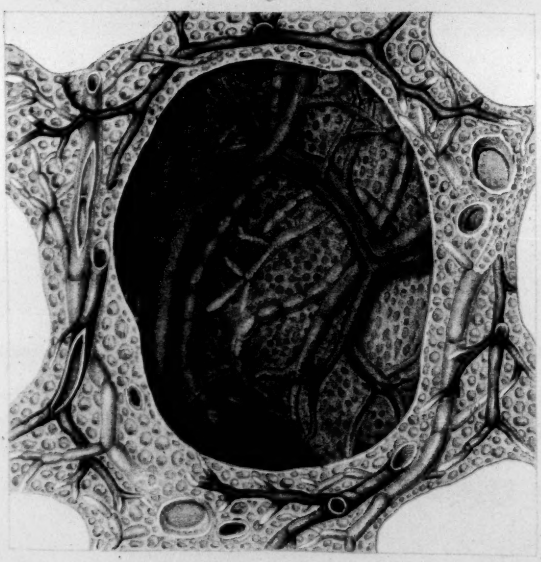
1

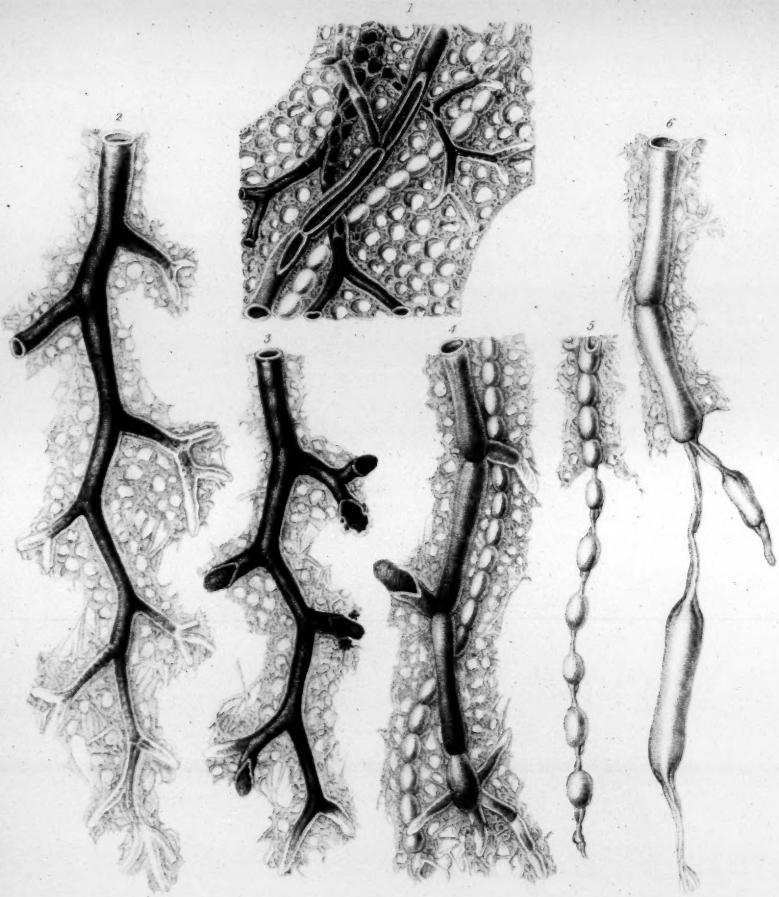


2



3




1
2
3
4
5
6

so that it is only such blood that is decarbonised by the carbonic acid that is carried off by expiration.

Respiration is necessary for carrying on the functions of life, by a supply of oxygen, and the removal of the excess of carbon; but it is not required for the simple support of the principle of life when no action is going on.

The garden snail excludes the atmospheric air from the lungs, by means of an operculum formed of mucus, during the winter season; and remains alive sealed up hermetically, till the warmth and the moisture of the spring dissolve the operculum, and expand a globule of air that had been shut up in the cavity of the lungs, the escape of which opens the communication with the outer air and restores respiration.

EXPLANATION OF THE PLATES.

Plate VIII. Shows the cells of the human lungs as they appear on the field of the microscope.

Fig. 1. represents $\frac{1}{64}$th part of an inch of the external surface of the human lung, the cells of which are filled with quicksilver; magnified 20 diameters.

Fig. 2. a transverse section of $\frac{1}{64}$th part of an inch of the human lung, in which the arteries are filled with red, and the veins with yellow minute injection; magnified 20 diameters.

Fig. 3. the transverse section of a single cell from fig. 2. with the parts immediately surrounding it; magnified 400 diameters.

Plate IX. Shows the structure of the human lungs immediately surrounding the air cells.

Fig. 1. the interstitial substance between the cells, in a

recent state, consisting of small cells every where opening into one another, and communicating with the large one; the arteries and veins injected; magnified 400 diameters.

Fig. 2. a portion of a terminal branch of the pulmonary artery at the air cell of the lungs, in which the injection has not reached the termination, and the remaining portion is distended with gas; magnified 400 diameters.

Fig. 3. a portion of an injected terminal branch of the pulmonary artery, into which the injection has been too much forced, which has occasioned extravasation through the terminating ends of the branches; magnified 400 diameters.

Fig. 4. a portion of an injected vein, accompanied by an absorbent vessel. At one terminating branch of the vein the injection has also been forced out; magnified 400 diameters.

Fig. 5. a portion of an absorbent vessel, in a dried state, in which some of the intervalvular portions are inflated, others collapsed; magnified 400 diameters.

Fig. 6. a portion of a vein dried, in which some of the intervalvular portions are distended with gas; magnified 400 diameters.

VII. *Remarks on a correction of the solar tables required by Mr.* SOUTH's *observations. By* G. B. AIRY, *Esq. M. A. Fellow of Trinity College, Cambridge, and Lucasian Professor of Mathematics in the University of Cambridge. Communicated by Dr.* YOUNG, *F. R. S. &c.*

Read February 15, 1827.

THE discordances between the sun's true ☊, as observed by Mr. SOUTH, and his calculated ☊, as given in the Nautical Almanac, follow a law so simple, as to justify us in attributing them principally to the errors of the solar tables. The only exceptions to this assertion are the differences of March 1st, December 22d, and December 23d, 1822; and upon examination of the sun's calculated ☊ for several days, previous and subsequent to those days, it appears that there is some irregularity in the second differences. I imagine therefore that some small errors have been accumulated in the calculations for those days; but as this is merely conjectural, I have not thought myself at liberty to reject them in the following computations.

A single inspection of the discrepancies, is sufficient to show that they are almost entirely produced by an error of the epoch, and an error in the place of the perigee. With these errors only I begun my calculations; but finding that the construction of the solar tables (contained in VINCE's Astronomy) gave great facilities for introducing an error in the eccentricity, I begun my calculations again, supposing the epoch, the place of the perigee, and the eccentricity, or the greatest equation

of the centre, liable to error. The error of the equation of the centre is found to be so small that it may be neglected: but the errors of the epoch and place of the perigee are considerable.

The first part of the operation was to deduce from the errors in $\mathfrak{R}$, the corresponding errors in longitude. This was done by multiplying them by 15 sec. 23° 28′ cos.° dec.; the multiplication by sec. 23° 28′ was however reserved to the end, when the results are multiplied by it. The next was, to give the errors which would be occasioned by assumed errors in the epoch, the place of the perigee, and the greatest equation of the centre. As the tables contain the variation of the equation of the centre for a variation of 10′ in the mean anomaly, and for one of $-17''{,}18$ in the greatest equation, this was very easily effected.

Supposing then that the epoch ought to be increased by x'', the mean anomaly by $y \times 10'$, and the greatest equation of the centre by $-z \times 17''{,}18$, I get the following equations, each of which is erroneous to the amount of the error of observation. The first side (as was mentioned) ought to be multiplied by sec. 23° 28′.

$$\begin{aligned}
10{,}57 &= x - y \times 19{,}7 - z \times 0{,}48 \\
11{,}87 &= x - y \times 19{,}6 + z \times 2{,}1 \\
10{,}38 &= x - y \times 19{,}4 + z \times 2{,}91 \\
13{,}87 &= x - y \times 19{,}1 + z \times 4{,}59 \\
12{,}8 &= x - y \times 18{,}9 + z \times 4{,}87 \\
10{,}31 &= x - y \times 16{,}9 + z \times 8{,}75 \\
8{,}94 &= x - y \times 16{,}7 + z \times 8{,}99 \\
12{,}49 &= x - y \times 15{,}6 + z \times 10{,}43 \\
10{,}76 &= x - y \times 15{,}4 + z \times 10{,}65 \\
8{,}43 &= x - y \times 14 + z \times 12
\end{aligned}$$

$$14{,}16 = x - y \times 13{,}4 + z \times 12{,}54$$
$$10{,}51 = x - y \times 13{,}1 + z \times 12{,}74$$
$$10{,}85 = x - y \times 12{,}8 + z \times 12{,}93$$
$$10{,}85 = x - y \times 12{,}6 + z \times 13{,}12$$
$$9{,}81 = x - y \times 12{,}3 + z \times 13{,}31$$
$$9{,}73 = x - y \times 9{,}9 + z \times 14{,}79$$
$$9{,}90 = x - y \times 9{,}6 + z \times 14{,}94$$
$$9{,}76 = x - y \times 9{,}3 + z \times 15{,}08$$
$$9{,}76 = x - y \times 9 + z \times 15{,}22$$
$$11{,}99 = x - y \times 6{,}8 + z \times 16{,}05$$
$$10{,}19 = x - y \times 5{,}8 + z \times 16{,}34$$
$$12{,}09 = x - y \times 5{,}5 + z \times 16{,}43$$
$$11{,}6 = x - y \times 2{,}8 + z \times 16{,}96$$
$$10{,}52 = x - y \times 2{,}5 + z \times 17,$$
$$11{,}93 = x - y \times 0{,}1 + z \times 17{,}18$$
$$10{,}57 = x + y \times 6{,}5 + z \times 16{,}38$$
$$9{,}25 = x + y \times 6{,}8 + z \times 16{,}3$$
$$7{,}51 = x + y \times 9{,}1 + z \times 15{,}51$$
$$8{,}6 = x + y \times 9{,}4 + z \times 15{,}37$$
$$7{,}85 = x + y \times 11{,}5 + z \times 14{,}32$$

$$7{,}43 = x + y \times 17{,}9 + z \times 8{,}59$$
$$8{,}01 = x + y \times 18{,}3 + z \times 8{,}05$$
$$5{,}43 = x + y \times 18{,}4 + z \times 7{,}77$$
$$7{,}22 = x + y \times 18{,}5 + z \times 7{,}5$$
$$5{,}78 = x + y \times 18{,}9 + z \times 6{,}98$$
$$6{,}39 = x + y \times 19{,}3 + z \times 6{,}11$$
$$6{,}21 = x + y \times 19{,}8 - z \times 4{,}53$$
$$6{,}54 = x + y \times 19{,}8 - z \times 4{,}78$$

$$8{,}74 = x + y \times 12{,}6 - z \times 13{,}7$$
$$5{,}52 = x + y \times 12 - z \times 14{,}07$$
$$8{,}45 = x + y \times 11{,}7 - z \times 14{,}24$$

$$3{,}31 = x + y \times 10{,}6 - z \times 14{,}87$$
$$0{,}44 = x + y \times 10{,}3 - z \times 15{,}01$$
$$10{,}04 = x - y \times 9{,}7 - z \times 14{,}87$$
$$9{,}87 = x - y \times 10 - z \times 14{,}72$$

$$11{,}39 = x - y \times 15{,}1 - z \times 10{,}99$$
$$12{,}19 = x - y \times 15{,}3 - z \times 10{,}77$$
$$7{,}84 = x - y \times 15{,}7 - z \times 10{,}31$$
$$12{,}81 = x - y \times 16{,}3 - z \times 9{,}6$$
$$12{,}54 = x - y \times 17 - z \times 8{,}62$$
$$10{,}65 = x - y \times 17{,}1 - z \times 8{,}41$$
$$10{,}62 = x - y \times 17{,}3 - z \times 8{,}16$$
$$11{,}22 = x - y \times 17{,}5 - z \times 7{,}9$$
$$9{,}02 = x - y \times 17{,}6 - z \times 7{,}64$$
$$11{,}85 = x - y \times 17{,}9 - z \times 7{,}12$$
$$9 = x - y \times 18 - z \times 6{,}86$$
$$8{,}76 = x - y \times 19{,}5 - z \times 2{,}8$$
$$11{,}19 = x - y \times 19{,}7 + z \times 0{,}6$$
$$13{,}99 = x - y \times 19{,}6 + z \times 1{,}44$$
$$12{,}51 = x - y \times 19{,}6 + z \times 2{,}29$$
$$12{,}01 = x - y \times 17{,}5 + z \times 7{,}94$$
$$9{,}08 = x - y \times 17{,}3 + z \times 8{,}2$$
$$11{,}03 = x - y \times 17{,}2 + z \times 8{,}45$$
$$10{,}03 = x - y \times 17 + z \times 8{,}7$$
$$9{,}53 = x - y \times 16{,}8 + z \times 8{,}95$$
$$11{,}53 = x - y \times 16 + z \times 9{,}88$$
$$11{,}98 = x - y \times 15{,}8 + z \times 10{,}15$$
$$9{,}43 = x - y \times 14{,}1 + z \times 11{,}9$$

$$10{,}57 = x - y \times 13{,}9 + z \times 12{,}1$$
$$8{,}03 = x - y \times 13{,}6 + z \times 12{,}31$$
$$8{,}25 = x - y \times 13{,}2 + z \times 12{,}68$$
$$9{,}31 = x + y \times 4 + z \times 16{,}9$$

$$6 = x + y \times 10{,}9 + z \times 14{,}69$$
$$5{,}05 = x + y \times 12{,}3 + z \times 13{,}86$$
$$4{,}65 = x + y \times 13{,}5 + z \times 13{,}09$$
$$5{,}85 = x + y \times 13{,}7 + z \times 12{,}89$$

$$4 = x + y \times 16{,}7 + z \times 10{,}2$$
$$6{,}07 = x + y \times 17{,}3 + z \times 9{,}43$$
$$6{,}96 = x + y \times 18{,}5 + z \times 7{,}59$$
$$6{,}72 = x + y \times 18{,}7 + z \times 7{,}31$$
$$5{,}05 = x + y \times 18{,}8 + z \times 7{,}02$$
$$2{,}07 = x + y \times 20{,}3 + z \times 2{,}9$$
$$2 = x + y \times 20{,}4 + z \times 2{,}6$$
$$4{,}7 = x + y \times 20{,}5 + z \times 1{,}72$$
$$3{,}53 = x + y \times 20{,}5 + z \times 1{,}09$$
$$6{,}23 = x + y \times 20{,}5 + z \times 0{,}47$$

Grouping together those equations in which the sun's anomaly is included between 1° 30° and 4° 30°, between 4° 30° and 7° 30°, between 7° 30° and 10° 30°, and between 10° 30° and 1° 30°, as marked by the divisions above, we have the following results.

Summer of 1821.

$$110{,}42 = 10\,x - y \times 175{,}3 + z \times 64{,}81.$$

Autumn of 1821.

$$207{,}43 = 20\,x - y \times 82{,}2 + z \times 302{,}51.$$

Winter of 1821-1822.

$$53{,}01 = 8\,x + y \times 150{,}9 + z \times 35{,}69.$$

Spring of 1822.

$$46{,}37 = 7\,x + y \times 37{,}5 - z \times 101{,}48.$$

Summer of 1822.

$$250{,}2 = 23\,x - y \times 394{,}9 - z \times 20{,}68.$$

Autumn of 1822.

$$57{,}71 = 8\,x + y \times 13{,}7 + z \times 108{,}52.$$

Winter of 1822.

$$47{,}33 = 10\,x + y \times 192{,}2 + z \times 50{,}33.$$

And adding the groups for the same seasons of the two years,

Observations in Spring give $46{,}37 = 7x + y \times 37{,}5 - z \times 101{,}48$

in Summer $- 360{,}62 = 33x - y \times 570{,}2 + z \times 44{,}13$

in Autumn $- 265{,}14 = 28x - y \times 68{,}5 + z \times 411{,}03$

in Winter $- 100{,}34 = 18x + y \times 343{,}1 + z \times 86{,}02$

Adding all,

$$772{,}47 = 86x - y \times 258{,}1 + z \times 439{,}7.$$

Subtracting the Winter from the Summer equations,

$$260{,}28 = 15x - y \times 913{,}3 - z \times 41{,}89.$$

Subtracting the Spring from the Autumn equations,

$$218{,}77 = 21x - y \times 106 + z \times 512{,}51.$$

By solving these equations we find $x = 8{,}23$, $y = -\frac{1}{6{,}555}$, $z = \frac{1}{17{,}18}$. These are to be multiplied by sec. 23°. 28′ = 1,0902. Performing this multiplication, and forming the quantities x'', $y \times 10'$, and $-z \times 17{,}''18$ we find that the epoch ought to be increased $8^{s}{,}97$, the mean anomaly ought to be diminished $99''{,}8$, and the greatest equation of the centre diminished $1''{,}09$. The epoch of the perigee ought therefore to be increased $107''{,}8$.

The correction of the equation of the centre is so small, that it may be doubtful whether it would be necessary to consider it. In that case, the solar tables would require no other alteration than in the tables of epochs. Every epoch of the sun must be increased by $8''{,}97$ or $9''$, and every epoch of the perigee by $1'{,}48''$.

If we reject the equation corresponding to the observations of March 1st, December 22d, and December 23d, 1822, (the 43d, 82d, and 83d of the preceding list), we find that the epoch of the sun's longitude must be increased by $9''{,}3$, and that of the perigee by $1'.39''$, and that the greatest equation of the centre must be diminished by $0''{,}66$.

VIII. *On the mutual action of the particles of magnetic bodies, and on the law of variation of the magnetic forces generated at different distances during rotation.* *By* S. H. CHRISTIE, *Esq. M. A. F. R. S.*

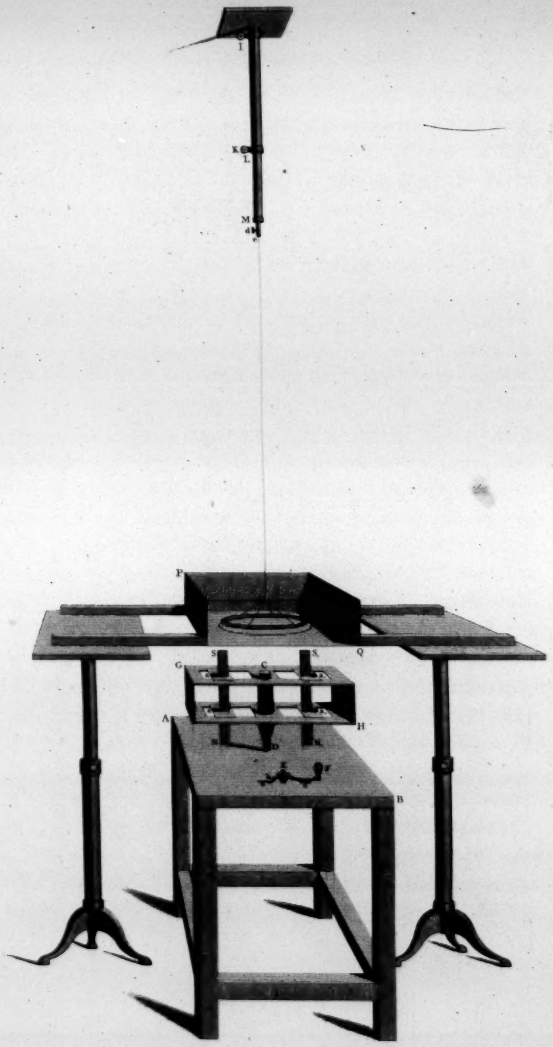
Read February 15 and 22, 1826.

IN my Letter to Mr. HERSCHEL, published in the Transactions, I communicated the results of some experiments which I had made, with the view of determining according to what law of distance the magnetism developed in copper and other metals during rotation, varied. I was aware, that all I could offer at that time must be considered as distant approximations towards useful results; but as I had witnessed several of the very interesting experiments in which Mr. HERSCHEL and Mr. BABBAGE were engaged, I was desirous that they should be in possession of whatever results I had obtained. In these experiments, I had made use of a thick copper plate revolving under a magnetized needle, and of magnets revolving under a copper disc; so that, at different distances, some of the forces which were brought into action during rotation, had very different angles of inclination to the plane of the needle, in the one case, and to the plane of the disc in the other; and in consequence of this, the results were by no means uniform. In order to remedy this, I proposed, instead of a disc, to make use of a copper ring, as, in this case, the poles of the magnet revolving vertically under it, no lateral forces would be called into action; and I expected that the

law, according to which the force acting during rotation varied at different distances, would be determined with great precision. In this expectation I was not disappointed; and I now propose laying before the Society an account of these experiments, and of the results which I obtained from them.

In making some preliminary trials with thin flat rings, I found the effects produced so very much less than with a disc of the same weight, that, previously to pursuing the enquiry which I had at first in view, I was induced to make some experiments, in order to ascertain the effects that would be produced by a simple solution of continuity, in a circular disc, by concentric sections, and likewise by successively removing concentric portions. These experiments clearly show, that the intensity of the magnetism developed during rotation, is not alone materially affected by a separation across, what may be termed, the path of the induced pole, as has been found to be the case by Mr. BABBAGE and Mr. HERSCHEL; but that a separation, concentric with that path, by which the pole is undisturbed in its progress, is equally efficacious in diminishing the intensity of that pole; and that in the magnetism of the whole, when all the parts are continuous, there is in all cases a great accumulation of intensity above the sum of the intensities of the separate parts. This is so important a feature in the phænomena depending upon rotation, that these experiments may not be considered uninteresting: I shall therefore give an account of them previously to entering upon the principal object of this communication.

The nature of the apparatus which I employed on this and subsequent occasions, will be best understood by a reference to Plate X. A B is a very firm table, having a vertical axis


I
K
L
M
P
Q
S
R
G
A
H
B

C D passing through it, to which a rapid rotation can be given by means of the handle E F, to the axis of which, under the table, a wheel is attached, having a band passing round the lower part of the axis C D. In the upper and under sides of the wooden frame G H, firmly screwed on the axis C D, are openings in which the brass frames a, a, a_1, a_1, slide, and can be clamped at any distance from the axis, by means of milled headed screws passing through them, and working in brass nuts, holding on each side of the slits. In the brass frames a, a, a_1, a_1, are square openings to admit the magnets S N, S_1 N_1, either with their faces or their edges towards the axis of rotation, and in which they can be firmly wedged. By this means the distance from the axis of rotation at which the magnets revolve, and likewise the verticality of their axes, can be very accurately adjusted. From the handle E F, a brass spring b projects, and this striking on two small elevations c, c_1, diametrically opposite to each other on the table, indicates very accurately the time of half a revolution. I K is a tube fixed to the ceiling of the room at I, and having a clamping screw at K, by which the tube L M, sliding within the other I K, can be fixed. The suspending wire $e f$ is nipped tightly in a piece of slit brass by means of the screw d. The lower end of the suspending wire is similarly fixed between two small flat pieces of brass, which, below, form a broad stirrup, into which small wires, from the angles of a light rectangular wooden scale, may be passed, and there crossing each other, any torsion of the suspending wire immediately acts upon the scale, and *vice versa*. From the suspended scale a small piece of brass projects, which indicates in circles and degrees, on a large graduated ring,

on the bottom of the screen P Q, whatever torsion is given to the wire, thus forming a very accurate balance of torsion. The screen P Q, having paper stretched tightly for the bottom and all the sides raised five inches, prevented currents of air, caused by the rotation of the magnets, affecting the disc or ring suspended over them. A B was so fixed that the axis C D, and the suspending wire *e f* were in the same vertical line, and with this line the centre of the graduated ring also coincided.

On the mutual action of the particles of magnetic bodies during rotation.

Having cut two discs of the same diameter from the same sheet of copper, I suspended one of them at the distance of an inch from the upper horizontal surfaces of the magnets, whose axes were vertical, and south poles upwards, and ascertained by means of this apparatus, the effects that would be produced by making the magnets revolve under it with their axes at various distances from the axis of rotation. I then observed the effect of making a circular cut through the disc, at the distance of an inch from its circumference; first, when a portion was left uncut; and next, when the outer ring was entirely separated from the remaining inner disc. The second disc was suspended by the same wire, and at the same distance from the magnets; and the effects which were produced, by making successively circular cuts, at different distances from the centre, ascertained, whether the interior portions were removed or retained.

The diameter of each of the discs was 8·4 inches; the weight of that which I designate as I. was 5298 grains; that

of II. was 5232 grains. I preferred making observations with discs having this difference in weight, to employing a file to reduce them to the same. In observing with the disc II. glass, weighing 66 grains, was added; and in every case where the weight of copper suspended was less than 5298 grains, discs of wood and of glass were added to make up that weight, in order that the tension of the suspending wire might be the same in all cases. This wire was of hard brass of the size called by the wire-drawers No. 22, its length 45·6 inches.

The disc I. being placed in the scale at the distance of one inch from the upper surfaces of two 12 inch magnets, these were adjusted in the frame G H, with their flat faces towards the axis of rotation, so that their axes were vertical, and at the distance 4·2 inches from the axis of rotation, or that these axes revolved directly under the edge of the disc. The tube L M was turned until there was no torsion in the wire when the index on the scale pointed to zero on the graduated ring. The magnets were now made to revolve in the direction of screwing with the angular velocity of 5 revolutions per second, corresponding to one revolution of the handle in two seconds, (which velocity was always carefully preserved in all the subsequent experiments,) and the time in which the disc performed one, two, three, &c. revolutions observed, until it began to revolve in a direction contrary to that of the magnets; the instant when this happened was noted, and likewise the number of revolutions of the index, and the degree where it pointed when it began to retrograde. The same was done when the magnets revolved in the direction of unscrewing, and the means in the two directions taken. Having no

assistance in making the experiments I could only note the time to the nearest second.

The numbers in the first column of the following table indicate the torsion of the suspending wire in circles, or the number of revolutions performed by the disc from rest; and in the following columns, are set down, the times in which these revolutions were performed when the magnets revolved at the several distances indicated above the respective columns.

Table I.

Distance of the axes of the magnets from the axis of rotation =		Inches 4·2	Inches 3·7	Inches 3·2	Inches 2·7	Inches 2·2	Inches 1·7
		Time.	Time.	Time.	Time.	Time.	Time.
Torsion of the wire in circles, or No. of revolutions of the disc	1	sec. 91·5	sec. 67·5	sec. 58·0	sec. 59·0	sec. 70·5	sec. 96·0
	2	139·0	99·0	87·0	88·0	103·5	147·0
	3	190·5	125·0	109·0	111·0	132·5	206·5
	4	.....	152·0	130·0	134·0	161·5	
	5	.....	180·5	151·0	158·0	194·5	
	6	.....	222·0	173·0	180·0	246·5	
	7	.....	.....	199·5	213·0		
	8	.....	.....	237·0			
Disc beginning to revolve in direction opposite to that of the magnets	Arc =	3⊖ 238°	6⊖ 152·5°	8⊖ 107°	7⊙ 268·5°	6⊖ 52°	3⊖ 121°
	Time =	sec. 267·0	sec. 269·5	sec. 270·5	sec. 269·5	sec. 272·5	sec. 270·0

In order to deduce from these times, the force with which the disc was urged by the magnets revolving at different distances from the centre, let us suppose that this force is equivalent to a certain force acting at the distance 1 from the centre of rotation, and that this latter force would balance a torsion α° of the wire, or that it is equal to $m\alpha$, m being a constant to be deduced from the experiments: also let t be the time in which the index on the scale describes an angle

θ, or in which the torsion of the wire is θ; and v the velocity of a point in the disc at the distance 1 from the axis of rotation; α, θ and v being in degrees to the radius 1. We have therefore,

$$v\,dv = m\,(\alpha - \theta).\,d\theta,$$

$$v^2 = m\,(2\,\alpha\,\theta - \theta^2),$$

$$\text{and } t = \frac{1}{\sqrt{m}}\,.\,\text{vers.}^{-1}\frac{\theta}{\alpha}.$$

Let the values of t and θ, when v becomes o, or the disc begins to turn back, be t_1 and θ_1, then we shall have

$$2\,\alpha\,\theta_1 - \theta_1^2 = o\,, \text{ and } \alpha = \tfrac{1}{2}\,\theta_1\,.$$

$$\text{also, } t_1 = \frac{1}{\sqrt{m}}\,.\,\pi\,, \text{ and } \sqrt{m} = \pi\,.\,\frac{1}{t_1}\,.$$

The mean value of t_1 from the experiments is 269·83 seconds, or very nearly 270 seconds; so that $\sqrt{m} = 2\,.\,\frac{1}{3}$.

We have therefore $\frac{\theta}{\alpha} = \text{vers}\,(2^\circ.\,\frac{t}{3})$; whence

$$\alpha = \frac{\theta}{\text{vers.}\,(2^\circ.\,\frac{t}{3})}.$$

The corresponding values of θ and t, contained in the preceding table, being substituted in this formula, will give the following values of α, or of the relative forces with which the magnets urged the disc when revolving at different distances from the axis.

Table II.

Distance of the axes of the magnets from the axis of rotation =		Inches 4·2	Inches 3·7	Inches 3·2	Inches 2·7	Inches 2·2	Inches 1·7
		α	α	α	α	α	α
Values of θ from which the values of α are deduced.	360	698·8	1229·1	1642·3	1589·2	1132·1	641·0
	720	688·0	1213·6	1531·6	1500·0	1122·0	632·0
	1080	676·2	1221·8	1538·3	1491·0	1112·3	620·9
	1440	.....	1203·5	1528·9	1457·0	1104·7	
	1800	.....	1196·0	1518·9	1423·6	1098·7	
	2160	.....	1168·8	1512·0	1440·0	1100·4	
	2520	.....	.....	1498·2	1409·4		
	2880	.....	.....	1494·4			
$\alpha = \frac{1}{2}\theta_1 =$		659·0	1156·3	1493·5	1394·3	1106·0	600·5
Mean values of α		680·5	1198·4	1528·7	1463·1	1110·9	623·6

The values of α deduced from the different values of θ, for the same distance of the magnets, agree with each other as nearly as could be expected, excepting in two cases, those corresponding to θ 360°, at the distances 3·2 inches and 2·7 inches, which are so much greater than the other values of α at those distances, that I am disposed to think the observations from which they are deduced inaccurate. A slight impulse given to the disc when it was released at zero, would diminish most sensibly the time of the first revolution, and consequently increase the corresponding value of α; and as it was difficult to avoid this in all cases, the inaccuracy in the observation might possibly arise from this cause. I was not aware of the incongruity in these values of α until it was too late to repeat the observations, having cut the disc and moved the apparatus before I made the computations.

Not only in these, but in all the observations which I have made, and from which I have computed the values of α, I have almost invariably found, that the values of α decrease as those of θ, from which they are deduced, increase: in 169 observations there are only 14 exceptions, and in these, in general, the differences are so small, that they most probably arose from small errors in the observations. It would appear then, that in all the experiments there must have been some circumstance which has a tendency to diminish the value of α as that of θ increased. It at first occurred to me that this might be a small deviation from the received law, in the connection between the force exerted by the wire and its torsion; but this I found was not the case, as the results which I obtained by comparing the deviations of a magnetized needle with the torsion of the wire, were extremely uniform. If the force

exerted by the wire be proportional to the torsion, the torsion divided by the sine of the deviation should be constant. Now the values which I obtained for this quotient with the torsions 713°·8, 1427°·5, 2141°·17, were 6600, 6595, 6634, the greatest difference among which is only the 165th part of the mean. The resistance of the air would have a tendency to increase the whole time during which the disc continued to revolve in the direction of the magnets, and consequently to diminish the values of α as those of θ increased, but not to a sufficient extent; a diminution of 6 or 7 seconds in the whole time, and in some instances more, being required to account for this decrease in the value of α: nor can the decrease arise from the small changes which take place in the relative velocity of the disc and the magnets. Upon the whole, I think it very probable, that the values of α really decrease in the successive revolutions of the disc; and in this manner: according to the principle adopted in the very interesting Paper by Mr. HERSCHEL and Mr. BABBAGE, *on the magnetism manifested by various substances during rotation*, and so well supported by all the phenomena hitherto observed, the effect of the magnets upon the disc will depend upon the excess of the magnetism in those parts of the disc in the rear of the magnets above that in the parts in advance of them; and if this excess were constant, so would also be the value of α; but if on the magnets coming successively under any point, that point has not parted with all the magnetism which it is capable of losing by the removal of the magnets to the opposite side of the disc, there may be a gradual, though small, accumulation of the magnetism left in every point of the disc; and as only a certain portion of magnetism can be developed in each point during the time that the

nets are in its vicinity, the excess which we have mentioned would be thus gradually diminished, and consequently also the value of α.

The forces with which the magnets, revolving at different distances with the same angular velocity urged the disc, are proportional to the mean values of α in Table II. and to deduce the relative forces with which the magnets would urge the disc when revolving at different distances with the same linear velocity, for instance, that at the distance 4·2 inches, the values of α must be increased in the inverse ratio of the respective distances, since the forces, *cæteris paribus*, are protional to the velocities. This reduction is made in the following Table.

Table III.

Distance of the axes of the magnets from the axis of rotation =	Inches. 4·2	Inches. 3·7	Inches. 3·2	Inches. 2·7	Inches. 2·2	Inches. 1·7
Force with which the magnets, revolving with the same linear velocity would urge the disc =	680·5	1360·3	2006·4	2275·9	2120·8	1540·7

So that the distance at which the magnets would produce the maximum effect by revolving with the same linear velocity would be very nearly 2·44 inches.

The distance from the axis at which the magnets must revolve, in order that the magnetism may be developed with the greatest intensity, is less than this; since, in estimating this intensity, the length of the lever, at the extremity of which the magnets act, must be taken into account. We shall obtain, at least approximately, the relative intensities of the nduced magnetism when the magnets revolved at different

distances from the axis, by increasing the numbers in Table III. in the inverse ratio of those distances, or by increasing the values of α in Table II. in the inverse ratio of the squares of the distances.

Table IV.

Distance of the Axes of the magnets from the Axis of rotation =	Inches. 4·2	Inches. 3·7	Inches. 3·2	Inches. 2·7	Inches. 2·2	Inches. 1·7
Intensity of the induced magnetism =	680·5	1544·1	2639·6	3540·3	4048·8	3806·4

It therefore appears from these experiments, that the intensity of the induced magnetism would have been the greatest had the magnets revolved at the distance 2·07 inches nearly; that is, so that their axes had been at the distance of half the radius of the disc from the centre. This is what we might expect, whatever may be the law according to which magnetism is developed in each particle; but the great diminution in the intensity when the magnets revolved under the edge of the disc, leads to an inference respecting the developement of magnetism by induction by no means unimportant, viz. that continuity is much more essential than mass; and that, although a certain portion of magnetism is developed in each particle separately, yet the whole of this is considerably less than that which appears to accumulate by the mutual action of particle upon particle; for which action continuity appears to be requisite.

The measure of the magnetism developed in the copper, if the magnets revolved at the distance 2·1 inches from the axis of rotation, would, interpolating roughly from the results in Table IV. appear to be 4182 nearly; so that 2091 would

nearly measure the quantity developed on each side of the axes of the magnets. The measure, therefore, of the magnetism developed when the magnets revolved with their axes under the edge of the disc, if continuity produced no accumulating effect, ought certainly not to be less than 2091 ; but as it is actually only 680·5, it would appear that the continuity of mass on each side of the axes of the magnets has the effect of increasing the quantity of magnetism developed at least in the ratio of 3 to 1.

The results which I obtained by making a circular cut in the disc, lead to a similar conclusion. In this case, the magnets were again adjusted to the distance 3·7 inches from the axis of rotation, and the times in which the disc completed successive revolutions was noted as before. A circular cut through the disc was now made at the distance of an inch from its edge, excepting for an inch at the extremities of two diameters at right angles to each other, as in fig. 1 ; after which the time at the completion of the revolutions was noted. The same was done when the disc was cut, as in fig. 2 ; and, finally, as in fig. 3 ; in which case tissue paper was placed between the ring cut off and the interior disc.

Fig. 1.

Fig. 2.

Fig. 3.

The results are arranged in the following Table :

Table V.

The magnets revolving with their axes at the distance 3·7 inches from the axis of rotation.

Torsion of the wire in circles, or No. of revolutions of the disc.	θ	Disc uncut. Time.	Disc uncut. α	Disc cut as Fig. 1. Time.	Disc cut as Fig. 1. α	Disc cut as Fig. 2. Time.	Disc cut as Fig. 2. α	Disc cut as Fig. 3. Time.	Disc cut as Fig. 3. α
		sec.		sec.		sec.		sec.	
1	360	67·0	1246·6	88·0	751·3	128·5	389·4	131·5	375·3
2	720	98·5	1224·5	132·0	746·0	217·5	395·8	236·0	374·5
3	1080	125·0	1221·8	178·5	727·4				
4	1440	153·0	1192·2						
5	1800	182·0	1184·0						
6	2160	224·0	1161·2						
Values of t, and $\alpha = \frac{1}{2}\theta^2$		269·5	1150·5	273·5	709·5	263·5	386·5	270·0	371·5
Mean values of α		.....	1197·3	.....	733·6	.....	390·9	.....	373·8

Here, the magnets revolving at the same distance in all cases, the mean values of α are measures of the magnetism developed under the different circumstances. The diminution of effect is sufficiently striking in the case of the ring remaining attached to the interior disc by four inches of its circumference, as in fig. 1. but it is still greater in proportion when the connection between them is further diminished by one half, as in fig. 2.; and no very striking effect, beyond this, is produced by rendering the separation complete. If any doubts could be entertained of the correctness of the inference respecting the effect of the absence of continuous matter, which I have drawn from the experiments when the magnets revolved at different distances from the axis under the entire disc, these last experiments, by exhibiting the effect itself in the most striking manner, precisely of the same nature and nearly in the same degree, must entirely remove them.

After having made these experiments, I ascertained what would be the effect produced when the magnets revolved under the separated ring alone, and likewise under the remaining disc alone: first, when the axes of the magnets were at the distance 3·7 inches from the axis of rotation, that is, remaining as in the foregoing experiments: secondly, when they revolved at the distance 3·2 inches from the axis, that is, having their axes under the inner edge of the ring, or the edge of the remaining disc.

Table VI.

The distance of the axes of the magnets from the axis of rotation.			
3·7 inches		3·2 inches.	
The magnets revolving under		The magnets revolving under	
The ring alone.	The disc alone.	The ring alone.	The disc alone.
$\alpha = 268{\cdot}0$	$\alpha = 120{\cdot}0$	$\alpha = 160{\cdot}5$	$\alpha = 283{\cdot}5$

The sum of the values of α, corresponding to the distance 3·7 inches, is rather greater than the value of α when the magnets revolved at the same distance under the disc and ring together; so that it would appear that no increase in the magnetism takes place in consequence of the proximity of the two masses. That the sum of the value of α, corresponding to the ring and disc separately, exceeds that of α corresponding to the ring and disc together, Table V. probably arises from a circumstance which affected slightly all the results. Although the direct communication between the disc and the currents of air, arising from the rotation of the magnets, was cut off by a screen of paper stretched on a frame, with sides considerably raised; yet a slight current of air was produced

near the disc, which increased the final arc of torsion 20° or 30°, or the values of α 10 or 15, as nearly as I could determine by making the magnets revolve under discs of wood and glass, of the same weight as the large copper disc. As I did not consider that the torsion could in all cases be determined within much less limits than this, I preferred giving the observations as they were made, to applying a correction which was doubtful in its amount.

When the magnets revolved at the distance 3·2 inches, the sum of the values of α is 444; and we may take this as the value of α when the magnets revolved under the disc and ring together at that distance. Now we have seen (Table II.) that when they revolved at the distance 3·2 inches from the axis under the uncut disc, the value of α was 1528·7; so that here the magnetism developed was diminished by the circular separation in the ratio of 3·44 to 1.

I now placed the disc II. in the scale, and having determined the effects that were produced by making the magnets revolve under it, with their axes at the distance of 3·2 inches from the axis of rotation, their upper surfaces being at the distance of an inch from the disc, and their angular velocity 5 revolutions in a second, the same as before; I determined successively the effects produced by making circular cuts at the distances ·7 inch, 1·2 inch, 1·7 inch, 2·2 inches, from the centre. I ascertained the effects both when the pieces in the interior were removed and when they were retained, excepting in the first case of the small disc, 1·4 inch in diameter, which was removed, and glass of the same weight substituted in the first experiment, but was afterwards replaced, with the rings cut out, in the others. The differences in the results

when the parts were retained, and when they were removed and glass substituted, were so small, that I shall only give the results where they were retained, excepting in the case of the largest circle. In all cases tissue paper was placed between the pieces to prevent contact.

Fig. 1. Fig. 2. Fig. 3. Fig. 4.

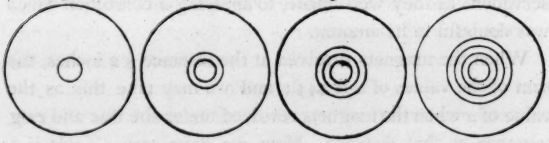


Table VII.

The magnets revolving with their axes at the distance 3·2 inches from the axis of rotation.

Torsion of the wire in circles, or No. of revolutions of the disc.	θ	Disc uncut. Time.	Disc uncut. α	Disc cut as Fig. 1. Time.	Disc cut as Fig. 1. α	Disc cut as Fig. 2. Time.	Disc cut as Fig. 2. α	Disc cut as Fig. 3. Time.	Disc cut as Fig. 3. α	Disc cut as Fig. 4. Time.	Disc cut as Fig. 4. α	Disc cut as Fig. 4, the interior rings and small disc removed. Time.	Disc cut as Fig. 4, the interior rings and small disc removed. α
		sec.		sec.		sec.		sec.		sec.		sec.	
1	360	58·0	1607·6	60·0	1506·1	63·0	1372·3	68·5	1170·2	79·0	896·3	78·5	907·1
2	720	85·5	1549·5	88·0	1468·6	92·5	1342·8	100·5	1158·8	116·5	898·6	116·5	898·6
3	1080	107·0	1558·0	111·0	1462·7	116·0	1357·6	126·5	1176·8	153·5	875.8	153·0	879·2
4	1440	128·0	1542·2	134·5	1423·4	140·5	1330·6	154·0	1163·5	194·5	868·7	194·0	871·0
5	1800	150·0	1517·7	157·3	1410·7	164·5	1326·7	184·5	1150·4				
6	2160	172·0	1502·5	183·5	1389·2	192·5	1317·3	223·0	1155·7				
7	2520	200·5	1474·4	220·0	1361·4	234·0	1308·7						
8	2880	252·5	1450·5										
Values of t, and $\alpha = \frac{1}{\theta}\theta_n$		267·0	1448·0	267·5	1352·0	265·5	1308·0	265·0	1116·8	266·0	855·5	268·5	864·0
Mean values of α		.....	1516·7	.....	1421·8	.....	1333·0	.....	1156·0	.....	879·0	.....	884·0

As might be expected, the effects produced by the circular separation are not great when the line of separation is near to the centre; but as it approaches the circumference under

which the magnets revolve, it increases rapidly. The portions of magnetism destroyed, or rather whose developement is prevented in the disc by circular cuts, at the distances ·7, 1·2, 1·7, 2·2 inches from the centre, appear, by these experiments, to be proportional to 94·9, 183·7, 360·7, 637·7. The last three numbers are very nearly proportional to the masses separated towards the centre, the quotients arising from dividing each by the square of the diameter of the section corresponding are 31·9, 31·2, 32·9; the mean 32·0; and the whole would have followed the same law, had the first number been 63 instead of 95, which difference is not greater than might take place in repetitions of the observations where the whole number of revolutions of the disc is 7 or 8, as in this case. When the magnets revolved under the disc I, with their axes at the distance 3·2 inches from the axis of rotation, the same as in the present instance, the value of α was 1528·7 (Table II.), and when they revolved at the same distance under the ring whose diameters were 6·4 inches, and 8·4 inches, its value was 160·5 (Table VI.); so that the quantity of magnetism whose developement was prevented by the removal of the disc diameter 6·4 inches, is proportional to 1368·2. Now, taking 32 as the quotient arising from dividing the number representing the quantity of magnetism destroyed, by the square of the diameter of the disc removed, 1310·7 will represent the quantity of magnetism destroyed by the removal of the disc diameter 6·4 inches. Supposing the law to be correct, the difference between this number, deduced from the observations with the disc 2, and the preceding, deduced from those with the disc 1, is not very far beyond the limits of errors in the observations, even had they,

in all the cases, been made with the same disc; and certainly the agreement is as near as we could expect from observations made with different discs.

If the agreement of all these results with so simple a law is to be considered as fortuitous, the coincidence is certainly most singular; but I am disposed to think that the law in nature is really this, that the quantities of magnetism destroyed by removing concentric portions from the interior of the disc, are proportional to the mass removed, but that they have not generally, the same ratio to the whole magnetism developed in the uncut disc, which the portion removed from the disc has to the whole disc; that is, if d is the diameter of the uncut disc, and α represent the magnetism developed in it; $d_{,}$ and $d_{,,}$ the diameters of portions removed, and $\alpha_{,}$, $\alpha_{,,}$ the magnetism developed in the disc when those portions are removed, the magnets revolving at the same distance from the axis in each case, then

$$\frac{\alpha - \alpha_{,}}{\alpha - \alpha_{,,}} = \frac{d_{,}^{2}}{d_{,,}^{2}},$$

and $\frac{\alpha - \alpha_{,}}{\alpha}$ is not equal to $\frac{d_{,}^{2}}{d^{2}}$, but $\frac{\alpha - \alpha_{,}}{\alpha} = q \cdot \frac{d_{,}^{2}}{d^{2}}$,

where q will be constant so long as the magnets revolve at the same distance from the axis, but will vary with that distance. In the present instance, the magnets revolving at the distance 3·2 inches from the axis, the values of q derived from the different observations are 1·48, 1·45, 1·53, 1·54, giving a mean 1·50. Here then the magnetism destroyed by the removal of concentric portions from the interior of the disc, has a considerably greater ratio to the whole magnetism developed than the mass removed has to the whole.

From the great diminution in the intensity of the induced

magnetism which takes place on making a circular cut in the disc, especially near to the magnets, it would follow, that if a copper disc were reduced to a series of concentric rings of small breadth, the magnetism developed in it would be scarcely appreciable under whatever part the magnets might revolve: the same would follow from the experiment of Mr. Babbage and Mr. Herschel, if a copper disc, by continual cutting in the direction of radii, were reduced nearly to a series of very small sectors; so that if a disc were separated in both directions, so as to be reduced to pieces of small magnitude, even though not in a state of powder, there can, I think, be no doubt that the induced magnetism would be rendered insensible.

The value of α (Table VII.) when the disc was cut, as in fig. 4. the rings and small disc being retained, is rather less than its value when they were removed: this, however, is only to be attributed to the almost unappreciable effect produced by the pieces in the interior, and unavoidable errors of observation. This will be evident from the results in the following table, obtained by placing an entire disc, 4·4 inches in diameter in the ring, instead of the small disc and rings. The observations were made some time after the preceding, when the apparatus had been moved, and the disc was not at *precisely* the same distance from the magnets, which was the reason of the effects being less than before. The magnets revolved at the same distance from the axis as before, 3·2 inches; first under the ring, diameters 4·4 inches, and 8·4 inches, with the disc 4·4 inches in diameter in the interior; then under the ring alone, and afterwards under the disc alone. The effect of even a slight current of air was excluded

by a glass cover over the screen, between the magnets and the disc.

Table VIII.

The magnets revolving with their axes at the distance 3·2 inches from the axis of rotation.								
			Under the ring, diameters 4·4 and 8·4 inches, with disc, diameter 4·4 inches.		Under the ring, diameters 4·4 and 8·4 inches, alone.		Under the disc diameter 4·4 inches alone.	
		θ	Time.	α	Time.	α	Time.	α
			sec.		sec.			
Torsion of the wire, or No. of revolutions of the disk.	1	360	82·0	819·0	84·5	785·0		
	2	720	123·0	806·1	127·0	770·2		
	3	1080	165·5	777·9	170·5	752·4		
	4	1440	226·5	757·1				
Values of t_1 and $\alpha = \frac{1}{2}\theta_1$			264·0	754·5	265·5	732·5	265·0	18·0
Mean values of α			.....	782·9	.....	760·0	.....	18·0

The sum of the mean values of α, when the magnets revolved under the ring alone, and under the disc alone, is here as nearly as can possibly be expected equal to the value of α when they revolved under the ring and disc together. If we attribute the whole difference to error in determining the value of α, when the magnets revolved under the disc alone, which however I have no reason for thinking should be done, the value of α would still in this case be only 23. Increasing this in proportion to the values of α in Table VII. the magnetism developed in the uncut disc, diameter 8·4 inches, that in the ring alone, diameters 8·4 and 4·4 inches, and that in the disc alone, diameter 4·4 inches, would very nearly be represented respectively by 1517, 880, and 26.

That the magnetism destroyed by the removal of a portion

of any mass should be more than a proportional part of the whole, does not appear extraordinary; but that the removal of a portion, in which, separately, the magnetism developed is represented by 26, should cause a diminution represented by 637 in the magnetism developed in the mass, or more than 24 times its separate effect; and that the separation should be nearly equivalent to the removal, are striking, and, I think, important facts in the investigation of the laws according to which magnetism is communicated to, and distributed in revolving bodies. These results are in perfect accordance with those which I have deduced from the experiments with the disc I.

It follows then from these experiments, that, in the developement of induced magnetism by rotation, every portion of a mass contributes towards the intensity of the magnetism developed, and that in a much greater ratio than would, by direct induction, be due to that portion according to its distance from the magnet; so that there appears to be, as it were, an accumulation of magnetism arising from the mutual action of the several particles upon each other; continuity throughout has thus a much greater influence than mass: a complete solution of continuity, when it does not take place in the parts adjacent to the magnet, is very nearly equivalent to an entire removal of the remoter mass so separated; and in all cases, the effect produced by complete separation is not much less than that produced by removal.

On the law of the variation of the magnetic forces generated at different distances during rotation.

I have already stated, that with the view of determining this law from experiment, I made use of a ring instead of a disc. This ring is of cast copper, circular, and of uniform thickness; its weight and dimensions as follow: weight 32·375 ounces troy; inner diameter 10·0 inches; outer diameter 11·9 inches; thickness ·24 inch. As I was desirous of avoiding using a file on it, the copper was simply cleared from the mould, and the small inequalities on its surface would consequently give all the measures in excess, so that its specific gravity is higher than 8·0954, which these dimensions would give. The mode which I adopted for suspending the ring was this:—an assemblage of four small wires (No. 18.) was fixed at the extremities of a diameter, so that when stretched, the middle was about two inches above the ring, and another similar assemblage fixed in the same manner at the extremities of the diameter at right angles: these were made to cross through the stirrup at *f*, as I have before described, so that there could be no play in it; that is, the ring could not be turned either way without immediately twisting the suspending wire. A double wire of the size No. 22, was attached to the stirrup, and after the ring had been suspended by it for several hours, its length was found to be 46·25 inches. To the ring, two pieces of wood, having small slips of brass projecting for indexes, were attached diametrically opposite to each other: these, with the wires fixed to the ring, made the whole weight supported by the suspending wire 32·56 ounces troy. The magnets before made use of, the dimensions of which

are, length 12·1 inches, breadth ·9 inch, thickness ·36 inch, were fixed vertically in the frame G H, equally distant from the axis of rotation, their edges towards that axis, their south poles upwards and upper surfaces horizontal. The distance between the inner edges of the magnets was 10·05 inches; between their outer edges 11·85 inches; so that their axes revolved exactly under the middle section of the ring, and they described a ring directly under the copper ring, of as nearly as possible the same horizontal dimensions.

The screen P Q being removed, the ring was lowered until it just touched without resting upon the upper surfaces of the magnets, when the distance between K and M, on the tube L M, was ascertained, 7·33 inches: adding ·12 inch to this, 7·45 inches would have been the distance between K and M, if the middle horizontal section of the ring could have coincided with the upper surfaces of the magnets. By subtracting in any case the distance between K and M from this number 7·45 inches, I had very accurately the distance between the upper surfaces of the magnets and the middle horizontal section of the copper ring. After finishing the experiments the screen P Q was removed, and the ring again lowered until, as before, it just touched the upper surfaces of the magnets, when the distance between K and M was found to be 7·34 inches, so that the wire could not have stretched during the experiments; the small difference ·01 inch must be attributed to slight inequalities on the surface of the copper.

The magnets being made, as before, to revolve 5 times in a second, in the direction of screwing, and preserving this velocity very carefully in all cases, by making the intervals between the beats of the spring, *b*, on c, c_1, exactly a second,

the times in which the ring completed successive revolutions, and in which it began to turn in the direction of unscrewing were noted; and also the number of revolutions and the degrees marked by the index when this took place. The same was done when the magnets revolved in the direction of unscrewing, and the means taken. The results obtained at different distances are tabulated below, where α, θ, θ_1, t, t_1, represent the same quantities as before.

Table I.

Distance of the middle section of the ring from the upper surfaces of the magnets.	Arc of torsion when the ring came to rest before retrograding. Values of θ_1	Then are $=\frac{1}{2}\theta_1$ when θ_1 was less than 360°, and are the mean of the values of α when θ_1 was greater than 360°. Values of α	Distance of the middle section of the ring from the upper surfaces of the magnets =		1·0 inch.		0·75 inch.		0·5 inch.	
				θ	t	$\alpha=\frac{\theta}{\text{vers. }\pi.\frac{t}{t_1}}$	t	$\alpha=\frac{\theta}{\text{vers. }\pi.\frac{t}{t_1}}$	t	$\alpha=\frac{\theta}{\text{vers. }\pi.\frac{t}{t_1}}$
					sec.		sec.		sec.	
			Torsion of the wire in circles, or No. of revolutions of the ring. 1	360	183·0	287·6	102·75	748·6	59·25	2188·3
			2	720	.....	.....	159·75	718·4	88·5	2032·1
			3	1080	.....	.....	219·0	685·5	110·125	2038·7
			4	1440	.....	.....	.....	.....	131·75	1983·4
			5	1800	.....	.....	.....	.....	151·62	1961·3
			6	2160	.....	.....	.....	.....	170·37	1960·7
			7	2520	.....	.....	.....	.....	190·75	1942·4
			8	2880	.....	.....	.....	.....	212·5	1929·1
			9	3240	.....	.....	.....	.....	236·25	1928·0
			10	3600	.....	.....	.....	.....	269·25	1916·5
			Values of t, and $\alpha=\frac{1}{2}\theta$,		315·0	290·75	315·0	672·5	320·25	1926·5
2·5 inches	22·75	11·375	Mean values of α		.....	289·18 (1)	.....	706·25 (2)	.....	1982·5 (3)
2·0	51·5	25·75								
1·5	144·0	72·00								
1·25	278·0	139·00								
1·00	581·5	289·18 (1)								
0·75	1344·5	706·25 (2)								
0·50 *	3853·0	1982·5 (3)								

* When the copper ring was at the distance ·5 inch from the magnets, its outer circumference was at the distance of less than ·25 inch from the inner circumference of the graduated ring, on which the deviations and revolutions of the copper ring were measured, and which rested upon the screen; but this produced no effect, at least that was sensible even at this small distance; for after removing the graduated ring, the times of completing the several revolutions of the copper ring, and likewise the whole arc of torsion before it came to rest, were found to be as nearly as possible the same as they were previous to the removal.

In order to obtain from these experiments the law according to which the force urging the ring at different distances varied, I assume that the whole action of the magnets may be referred to a single point, near the extremity in each, as a pole; and I consider that, within certain limits of the distances of the point acted on, no sensible error will arise from this supposition. My first trials were to ascertain whether this force varied inversely as any exact power of the distance, and I found that the supposition that it varied inversely as the 4th power of the distance, would give results approximating very closely to those obtained by observation. As however, on the principle that time is requisite both for the developement, and for the dissipation of magnetism, the induced poles in the ring would always be in the rear of the magnets, it appeared probable, that the supposition of the force varying inversely as some power of the distance between the pole of each magnet, and a point in the ring at a certain distance in the rear of it, would give results approximating even more closely to the observations; and on trial I found that this was decidedly the case.

Calling c the vertical distance of the upper surface of the magnets from the middle horizontal section of the ring, p the distance of the poles of the magnets from their upper surfaces, and s a constant horizontal distance from the pole of either magnet to a point behind it in the ring, I found that the formula,

$$\alpha = \left\{ \frac{M}{(p+c)^2 + s^2} \right\}^2 \ldots\ldots (1),$$

where M and p are constants which, with s, are to be determined from the observations, gave results approximating much more closely to the observations than the formula,

$$a = \left\{ \frac{M_1}{p_1 + c} \right\}^4.$$

That an estimate may be formed of the degree of coincidence, I shall give the comparison of the observations with the results obtained in both cases. Putting $\frac{1}{a^{\frac{1}{4}}} = a$ in the equation (1) we have

$$(p + c)^2 + \varepsilon^2 = M a \quad . \quad . \quad . \quad . \quad (2).$$

If we indicate the values of c and a, in the successive observations, by c_1, c_{11}, c_{111}, &c. and a_1, a_{11}, a_{111}, &c. and eliminate p and ε from three equations of the form (2), then

$$M = \frac{(c_l - c_m) \cdot (c_l - c_n) \cdot (c_m - c_n)}{a_n (c_l - c_m) - a_m (c_l - c_n) + a_l (c_m - c_n)} \quad . \quad (3).$$

Since, in determining the value of M from this formula by means of the observations, errors of observation to the same extent would be the more sensible the less the intervals $c_l - c_m$, $c_l - c_n$, $c_m - c_n$, between the observations, instead of employing all the possible combinations of the observations, in order to deduce the mean value of M, I have excluded all combinations in which two consecutive observations entered. This however makes but a very small difference in the results, since the mean value of M deduced from the thirty-five combinations of the seven observations is 23·362, and its value deduced from the ten combinations in which consecutive observations are excluded is 23·314.

The mean value of M being obtained, the value of p will be found by eliminating ε from two equations of the form (2), and we have thus,

$$p = \tfrac{1}{2} \left\{ \frac{M (a_l - a_m)}{c_l - c_m} - (c_l + c_m) \right\} \quad . \quad . \quad . \quad . \quad (4).$$

In computing the mean value of p from the different combinations of the observations, I, as before, exclude the combinations of consecutive observations.

The values of ε^2 are computed from the several equations of the form (2), by substituting the mean values of M and p; and I thus obtain the mean value of ε^2.

The following table exhibits the values of a in the different observations, the several values of M and p deduced from the different combinations of the observations, and the values of ε^2 derived from the separate observations.

Table II.

Values of c in different observations.	Values of a in different observations.	Values of M determined from different combinations of $a_{\mathrm{I}}, a_{\mathrm{II}}, a_{\mathrm{III}}, a_{\mathrm{IV}}$, &c.		Values of p determined from the mean value of M and different combinations of $a_{\mathrm{I}}, a_{\mathrm{II}}, a_{\mathrm{III}}$, &c.		Values of ε^2 determined from the mean values of M and p, and different values of a.
		Combinations.	Values of M.	Combinations.	Values of p	
inches.						
$c_{\mathrm{I}} = 2{\cdot}5$	$a_{\mathrm{I}} = {\cdot}296500$	$a_{\mathrm{I}}\ a_{\mathrm{III}}\ a_{\mathrm{V}}$	24·770	$a_{\mathrm{I}}\ a_{\mathrm{III}}$	·0825	·1255
$c_{\mathrm{II}} = 2\ 0$	$a_{\mathrm{II}} = {\cdot}197061$	$a_{\mathrm{I}}\ a_{\mathrm{III}}\ a_{\mathrm{VI}}$	24·412	$a_{\mathrm{I}}\ a_{\mathrm{IV}}$	·0990	·1625
$c_{\mathrm{III}} = 1{\cdot}5$	$a_{\mathrm{III}} = {\cdot}117851$	$a_{\mathrm{I}}\ a_{\mathrm{III}}\ a_{\mathrm{VII}}$	24·022	$a_{\mathrm{I}}\ a_{\mathrm{V}}$	·0972	·1709
$c_{\mathrm{IV}} = 1{\cdot}25$	$a_{\mathrm{IV}} = {\cdot}084819$	$a_{\mathrm{I}}\ a_{\mathrm{IV}}\ a_{\mathrm{VI}}$	23·344	$a_{\mathrm{I}}\ a_{\mathrm{VI}}$	·0994	·1409
$c_{\mathrm{V}} = 1{\cdot}00$	$a_{\mathrm{V}} = {\cdot}058805$	$a_{\mathrm{I}}\ a_{\mathrm{IV}}\ a_{\mathrm{VII}}$	23·202	$a_{\mathrm{I}}\ a_{\mathrm{VII}}$	·0973	·1495
$c_{\mathrm{VI}} = 0{\cdot}75$	$a_{\mathrm{VI}} = {\cdot}037629$	$a_{\mathrm{I}}\ a_{\mathrm{V}}\ a_{\mathrm{VII}}$	23·318	$a_{\mathrm{II}}\ a_{\mathrm{IV}}$	·1195	·1459
$c_{\mathrm{VII}} = 0{\cdot}50$	$a_{\mathrm{VII}} = {\cdot}022459$	$a_{\mathrm{II}}\ a_{\mathrm{IV}}\ a_{\mathrm{VI}}$	22·614	$a_{\mathrm{II}}\ a_{\mathrm{V}}$	·1117	·1573
		$a_{\mathrm{II}}\ a_{\mathrm{IV}}\ a_{\mathrm{VII}}$	22·553	$a_{\mathrm{II}}\ a_{\mathrm{VI}}$	·1118	
		$a_{\mathrm{II}}\ a_{\mathrm{V}}\ a_{\mathrm{VII}}$	22·878	$a_{\mathrm{II}}\ a_{\mathrm{VII}}$	·1069	
		$a_{\mathrm{III}}\ a_{\mathrm{V}}\ a_{\mathrm{VII}}$	22·026	$a_{\mathrm{III}}\ a_{\mathrm{V}}$	·1266	
				$a_{\mathrm{III}}\ a_{\mathrm{VI}}$	·1219	
				$a_{\mathrm{III}}\ a_{\mathrm{VII}}$	·1120	
				$a_{\mathrm{IV}}\ a_{\mathrm{VI}}$	·1002	
				$a_{\mathrm{IV}}\ a_{\mathrm{VII}}$	·0942	
				$a_{\mathrm{V}}\ a_{\mathrm{VII}}$	·0974	
		Mean	23·314	Mean	·1052	·1504

The differences here, between some of the values of M and p, and their mean values, may appear more considerable than they ought to be, if the formula from which they are derived be correct; but it is to be observed, that even very small errors in the observations are rendered very sensible by thus combining them. With regard to the differences in the values of ε^2, supposing the formula to be correct, and the observations to have been liable to no other errors than those in estimating the distances between the magnets and the ring, an error in this respect of $\frac{1}{100}$ inch would cause a greater difference than any here between the different values of ε^2 and the mean. As I took every precaution to ensure accuracy, I consider that although I might be liable to an error to this amount, I was not liable to a greater. The best criterion of the correctness of the formula is, however, the agreement of the observed values of α with those deduced from it by employing these mean values of M, p, and ε^2, and likewise the agreement in the values of these quantities deduced from the separate observations. The comparison between the observed values of α and those computed from the formula,

$$\alpha = \left\{\frac{M}{(p+c)^2+\varepsilon^2}\right\}^2,$$

assuming $M = 23{\cdot}314$, $p = {\cdot}1052$ and $\varepsilon^2 = 1504$, is made in the following table.

Table III.

Values of c in the different observations.	Observed values of α	Values of α computed from $\alpha = \left\{\frac{M}{(p+c)^2+\varepsilon^2}\right\}^2$	Difference between the observed & computed values of α.	Quotient of this diff. divided by the computed value of α.	Values of the constants deduced from the separate observations.		
					M	p	ε^2
inches.							
2·5	11·375	11·293	— 0·082	— ·00726	23·398	·1005	.1504
2 0	25·75	25·886	+ 0·136	+ ·00525	23·253	·1088	.1504
1·5	72·00	73·086	+ 1·086	+ ·01486	23·140	·1115	.1504
1·25	139·00	137·68	— 1·32	— ·00959	23·426	·1017	.1504
1 00	289·18	288·79	— 0·39	— ·00135	23·330	·1048	.1504
0·75	706 25	699·03	— 7·22	— ·01033	23·434	·1026	.1504
0·50	1982·5	2035·9	+ 53·4	+ ·02623	23·006	·1109	.1504
The mean values of M and p from the separate observations					23·284	·1058	

The quotients in the 5th column afford the best estimate of the degree of coincidence between the observed and computed values of α, since the magnitude of the errors to which the observations are liable depends upon the magnitude of α. The greatest discordance between the observed and computed values of α, whether estimated by their actual difference, or by dividing this difference by the computed value of α, is that corresponding to the least distance of the ring from the magnets; but even this amounts to little more than a fortieth of the whole. Perhaps it would have been as well to have omitted this observation altogether, since the distance of the ring from the magnets was so small, that supposing that distance to have been ·51 inch instead of ·50, or that an error of ·01 was made in estimating it, the difference between the observed and computed values of α would have been $-$ 39·4 instead of $+$ 53·4. Having however made the observations with great care, from this close agreement between the values of α deduced from the formula,

$$\alpha = \left\{ \frac{M}{(p+c)^2 + i^2} \right\}^2,$$

and those observed, I cannot but conclude that this formula is correct.

If we were to suppose that

$$\alpha = \left\{ \frac{M'}{p+c} \right\}^4;$$

then

$$M' = \frac{c_l - c_m}{a_l - a_m},$$

where $a_l = \left(\frac{1}{\alpha_l}\right)^{\frac{1}{4}}$, $a_m = \left(\frac{1}{\alpha_m}\right)^{\frac{1}{4}}$.

Combining the observations as I have already mentioned, I obtained the mean values $M' = 5{\cdot}060$ and $p = {\cdot}240$. From

the mean value of M′, deduced by combining all the observations, I had previously found $p = \cdot 256$, and $M' = 5 \cdot 1263$ from the separate observations, but I afterwards discovered that the latter values were slightly affected by an error in one of the computations; as however I had previously determined the values of α by means of them, and these are rather nearer to the observed values than those obtained from $M' = 5 \cdot 060$ and $p = \cdot 240$, I give the results of both to show the effects of such changes in the constants.

Table IV.

Values of c in the different observations.	Observed values of α	Values of α computed from $\alpha = \left\{\frac{M'}{p+c}\right\}^4$ $M' = 5 \cdot 060$ $p = 0 \cdot 240$	Difference between the observed & computed values of α	Quotient of this diff. divided by the computed value of α	Values of α computed from $\alpha = \left\{\frac{M'}{p+c}\right\}^4$ $M' = 5 \cdot 1263$ $p = 0 \cdot 256$.	Difference between the observed & computed values of α	Quotient of this diff. divided by the computed value of α
Inches.							
2·5	11·375	11·631	+ 0·255	+ ·02193	11·970	+ 0·595	+ ·04971
2·0	25·75	26·038	+ 0·288	+ ·01106	26·659	+ 0·909	+ ·03410
1·5	72·00	71·516	— 0·484	— ·00677	72·629	+ 0·629	+ ·00866
1·25	139·00	133·00	— 6·000	— ·04511	134·248	— 4·752	— ·03540
1·00	289·18	277·28	— 11·90	— ·04292	277·490	— 11·69	— ·04215
0·75	706·25	682·43	— 23·82	— ·03490	674·24	— 32·01	— ·04748
0·50	1982·5	2186·1	+ 203·6	+ ·09313	2114·1	+ 131·6	+ ·06226

The quotients in the 5th and 8th columns are not in any case great, but are decidedly greater than the corresponding quotients in Table III. Considering these quotients as the errors arising from the respective formulas, and that the sum of the squares of the errors is the best criterion of correctness, it appears that the sum of the squares of these errors in the 5th and 8th columns respectively, are ·014417 and ·012869, and that the sum of the squares of the errors in the 5th column of Table III. is only ·001190, or not a tenth of the

least sum in the other case. It is manifest then, that the introduction of the quantity ε into the expression for the value of α, renders it essentially more accurate; and the result of these different comparisons is, I think, quite decisive of the correctness of the formula,

$$\alpha = \left\{ \frac{M}{(p+c)^2 + \varepsilon^2} \right\}^2.$$

I have stated in my letter to Mr. HERSCHEL,* that when a thick copper plate revolved under a needle, the force by which the plate urged the needle appeared to vary nearly as the inverse *4th power* of the distance, but that when magnets revolved horizontally under a copper disc, the force with which they urged it appeared to vary according to a law approximating rather towards that of the inverse *square* of the distance. The difference of the law in the two cases arose no doubt from a light copper disc having been suspended over strong bar magnets, in the second experiment, instead of suspending the heavy copper plate over the needle used in the first experiment, which could not have been conveniently done, as its weight is about 16 lbs. but which would have reversed the experiment. To remove any doubt that might arise from this apparent incongruity, I determined now precisely to reverse the experiment in the two cases, making the copper ring revolve *under* the same magnets, *over* which it had been before suspended. As however I could not suspend the magnets vertically over the ring, without increasing so considerably the weight to be suspended, by the apparatus to fix them in, that the wire by which the ring had been suspended would no longer sustain them, and

* Phil. Trans. 1825.

indeed that no wire, on the torsion of which any sensible effect could be produced, would do so, the preceding experiments could not be made use of as strictly comparative with others in which the ring should revolve under the magnets. It therefore became necessary to make a different adjustment of the magnets, in the first instance, under the copper.

The magnets used in the preceding experiments were now placed horizontally, and parallel to each other, on their flat sides, on the top of the frame G H, with their poles of the same name adjacent, and their nearest sides at the distance ·34 inch from each other. The copper ring remained suspended by the same wire as in the preceding experiments, and being lowered until its under surface just touched, without resting upon two brass nuts, fixing the magnets to the frame, the distance between K and M was ascertained to be 9·575 inches: so that the distance from the under surface of the magnets to the under surface of the ring being ·59 inch, the thickness of the ring ·24 inch, and that of the magnets ·36 inch, the distance between K and M would have been 10·105, could the middle horizontal section of the ring have coincided with the axes of the magnets; which distance was therefore in this case to be considered as zero, on the scale measuring the distances between the middle horizontal section of the ring, in any case, and the horizontal plane passing through the axes of the magnets.

I have before mentioned, that, from the formula $a = \left\{\frac{M'}{p+c}\right\}^4$, I had found p, or the distance of the poles of the magnets from their extremities, to be ·256 inch, which, as I was not aware

at the time that any other formula would more closely represent the observations, I considered to be a near approximation, and that consequently this value of p was correct: and as I wished to see how far the effects produced by the magnets revolving with their axes vertical, agreed with the effects when they revolved with their axes horizontal, and their poles at the same distances from the ring, I regulated the distances at which I observed, in the latter case, according to this value of p. This will account for the peculiar distances at which I made observations which might otherwise appear somewhat singular. The results which I obtained are arranged in the following table.

Table V.

Distance of the middle section of the ring from the axes of the magnets.	Arc of torsion when the ring came to rest before retrograding. Values of θ_1	These are $= \frac{1}{2}\theta_1$ when θ_1 was less than 360°, and are the mean of the values of α when θ_1 was greater than 360°. Values of α	Distance of the middle section of the ring from the axes of the magnets. =		1·506 inch.		1·256 inch.		1·006 inch.		0·756 inch.	
				θ	t	$\alpha = \frac{\theta}{\text{vers. } \pi \cdot \frac{t}{t_1}}$	t	$\alpha = \frac{\theta}{\text{vers. } \pi \cdot \frac{t}{t_1}}$	t	$\alpha = \frac{\theta}{\text{vers. } \pi \cdot \frac{t}{t_1}}$	t	$\alpha = \frac{\theta}{\text{vers. } \pi \cdot \frac{t}{t_1}}$
			Torsion of the wire in circles, or No. of revolutions of the ring. 1	360	sec. 185·4	283·4	sec. 131·7	485·0	sec. 95·3	880·0	sec. 64·5	1873·2
			2	720	.....	.....	210·2	480·9	141·3	876·8	93·4	1853·4
			3	1080	.....	.....	.....	.....	184·3	870·1	117·3	1836·0
			4	1440	.....	.....	.....	.....	233·0	866·6	138·8	1829·1
			5	1800	.....	.....	.....	.....	.....	.....	160·0	1822·6
			6	2160	.....	.....	.....	.....	.....	.....	181·1	1803·1
			7	2520	.....	.....	.....	.....	.....	.....	203·8	1789·0
			8	2880	.....	.....	.....	.....	.....	.....	228·9	1779·8
			9	3240	.....	.....	.....	.....	.....	.....	261·5	1767·4
inches.												
2·756	71·0	35·5										
2·256	146·0	73·0										
1·756	342·5	171·25										
		(1)										
1·506	563·0	282·45										
		(2)										
1·256	956·0	481·3	Values of t, and $\alpha = \frac{1}{2}\theta$,		317·5	281·5	317·3	478·0	319·0	862·5	321·5	1763·75
		(3)										
1·006	1725·0	871·2										
		(4)				(1)		(2)		(3)		(4)
0·756	3527·5	1811·7	Mean values of α		.....	282·45	.....	481·3	.....	871·2	.....	1811·7

In the position which the magnets had here with regard to

the ring, the points to which we may refer all the forces of the magnets, that is their poles, will not be at the same distance from their extremities as when the magnets were vertical; and the errors which will arise from considering that these points continue fixed for different distances of the ring, will be increased when those distances are small, in consequence of the great obliquity of the direction in which some of the forces are exerted. The pole of the magnet will no longer be directly under the ring; but if we call c the vertical distance of the middle section of the ring from the axis of the compound magnet, and ε, as before, a constant horizontal distance from the pole, at either end, to a point behind it in the ring, then the formula (1) will be

$$\alpha = \left\{\frac{M}{c^2 + \varepsilon^2}\right\}^2 \qquad (5).$$

Putting, as before, a for $\frac{1}{\alpha^{\frac{1}{3}}}$, and calling the values of a corresponding to the distances c_l, c_m, a_l, a_m,

$$M = \frac{(c_l + c_m) \cdot (c_l - c_m)}{a_l - a_m} \qquad (6).$$

Combining these observations in the same manner as those which precede, I obtain the mean values of M, and then the values of ε^2 from the separate observations, by means of the formula,

$$\varepsilon^2 = M\,a - c^2 \qquad (7):$$

or we may obtain ε^2 independently of M from (5), then

$$\varepsilon^2 = \frac{c_l^2 a_m - c_m^2 a_l}{a_l - a_m},$$

or,

$$\varepsilon^2 = \frac{(c_l^2 - c_m^2) \cdot a^m}{a_l \quad a_m} - c_m^2 \qquad (8),$$

which latter is more convenient for computation; and the values of M may be computed from the separate observations.

The values of M computed from the formula (6), of ε^2 deduced from the separate observations with the mean value of M thus obtained, and also of ε^2 from the formula (8), are contained in the following table.

Table VI.

Values of c in different observations.	Values of a in different observations.	Values of M determined from different combinations of $a_{\mathrm{I}}, a_{\mathrm{II}}, a_{\mathrm{III}}, a_{\mathrm{IV}}$, &c.		Values of ε^2 determined from M = 48·518, and the separate observations.	Values of ε^2 determined from different combinations of $a_{\mathrm{I}}, a_{\mathrm{II}}, a_{\mathrm{III}}, a_{\mathrm{IV}}$, &c.	
		Combinations.	Values of M		Combinations.	Values of ε^2
inches.						
$c_{\mathrm{I}} = 2{\cdot}756$	$a_{\mathrm{I}} = {\cdot}167836$	$a_{\mathrm{I}}\ a_{\mathrm{III}}$	49·354	·5476	$a_{\mathrm{I}}\ a_{\mathrm{III}}$	·6880
$c_{\mathrm{II}} = 2{\cdot}256$	$a_{\mathrm{II}} = {\cdot}117041$	$a_{\mathrm{I}}\ a_{\mathrm{IV}}$	49·177	·5891	$a_{\mathrm{I}}\ a_{\mathrm{IV}}$	·6580
$c_{\mathrm{III}} = 1{\cdot}756$	$a_{\mathrm{III}} = {\cdot}076416$	$a_{\mathrm{I}}\ a_{\mathrm{V}}$	49·225	·6241	$a_{\mathrm{I}}\ a_{\mathrm{V}}$	·6663
$c_{\mathrm{IV}} = 1{\cdot}506$	$a_{\mathrm{IV}} = {\cdot}059502$	$a_{\mathrm{I}}\ a_{\mathrm{VI}}$	49·147	·6189	$a_{\mathrm{I}}\ a_{\mathrm{VI}}$	·6531
$c_{\mathrm{V}} = 1{\cdot}256$	$a_{\mathrm{V}} = {\cdot}045582$	$a_{\mathrm{I}}\ a_{\mathrm{VII}}$	48·662	·6340	$a_{\mathrm{I}}\ a_{\mathrm{VII}}$	·5725
$c_{\mathrm{VI}} = 1{\cdot}006$	$a_{\mathrm{VI}} = {\cdot}033880$	$a_{\mathrm{II}}\ a_{\mathrm{IV}}$	49·036	·6318	$a_{\mathrm{II}}\ a_{\mathrm{IV}}$	·6497
$c_{\mathrm{VII}} = 0{\cdot}756$	$a_{\mathrm{VII}} = {\cdot}023494$	$a_{\mathrm{II}}\ a_{\mathrm{V}}$	49·147	·5684	$a_{\mathrm{II}}\ a_{\mathrm{V}}$	·6627
		$a_{\mathrm{II}}\ a_{\mathrm{VI}}$	49·031		$a_{\mathrm{II}}\ a_{\mathrm{VI}}$	·6458
		$a_{\mathrm{II}}\ a_{\mathrm{VII}}$	48·297		$a_{\mathrm{II}}\ a_{\mathrm{VII}}$	·5631
		$a_{\mathrm{III}}\ a_{\mathrm{V}}$	48·842		$a_{\mathrm{III}}\ a_{\mathrm{V}}$	·6488
		$a_{\mathrm{III}}\ a_{\mathrm{VI}}$	48·700		$a_{\mathrm{III}}\ a_{\mathrm{VI}}$	·6378
		$a_{\mathrm{III}}\ a_{\mathrm{VII}}$	47·466		$a_{\mathrm{III}}\ a_{\mathrm{VII}}$	·5455
		$a_{\mathrm{IV}}\ a_{\mathrm{VI}}$	49·020		$a_{\mathrm{IV}}\ a_{\mathrm{VI}}$	·6487
		$a_{\mathrm{IV}}\ a_{\mathrm{VII}}$	47·114		$a_{\mathrm{IV}}\ a_{\mathrm{VII}}$	·5356
		$a_{\mathrm{V}}\ a_{\mathrm{VII}}$	45·545		$a_{\mathrm{V}}\ a_{\mathrm{VII}}$	·4985
		Means	48·518	·6020		

Assuming $M = 48{\cdot}518$, and $\epsilon^2 = {\cdot}6020$, the values of α corresponding to the different values of c may be computed from the formula (5). The comparison between the values of α, thus computed, and those observed, is made in the following table.

Table VII.

Values of c in the different observations.	Observed values of α	Values of α computed from $\alpha = \left\{\frac{M}{c^2+\epsilon^2}\right\}^2$	Diff. between the observed and computed values of α.	Quotient of this diff. divided by the computed value of α.	Values of M deduced from the separate observations.
inches					
2·756	35·5	35·03	— 0·47	—·01342	48·842
2·256	73·0	72·67	— 0·33	—·00454	48·628
1·756	171·25	173·31	+ 2·06	+·01189	48·229
1·506	282·45	285·79	+ 3·34	+·01171	48·234
1·256	481·3	495·56	+ 14·26	+·02878	47·814
1·006	871·2	903·65	+ 32·45	+·03591	47 639
0·756	1811·7	1709·4	—102·3	—·05985	49·949
Mean value of M from the separate observations - -					48·476

The agreement of the computed with the observed values of α, which I estimate by the quotients in the 5th column, is, upon the whole, as near as we could expect with this adjustment of the magnets. In the last observation, where the distance between the surfaces of the ring and magnets was only ·456 inch, the forces of some points in the magnets were exerted in directions so much more oblique than in the other observations, that this observation scarcely admits of comparison with the others, without taking into account the effect which this obliquity in the direction of the forces will have on the situation of the pole of the magnets. If in the comparison we reject this observation, the agreement between the computed and observed values of α becomes extremely

close. Omitting in the values of ε^2, in the 7th column of Table VI. all those in which a_{VII} enters, we shall have the mean value of $\varepsilon^2 = \cdot 6559$; and the mean value of M deduced from the separate observations will be 49·093. The values of α at the different distances, computed from these values of M and ε^2, are compared with the observed values of α, in the following table.

Table VIII.

Values of c in the different observations.	Observed values of α	Values of α computed from $\alpha = \left\{\frac{M}{c^2+\varepsilon^2}\right\}^2$	Diff. between the observed and computed values of α.	Quotient of this diff. divided by the computed value of α.	Values of M deduced from the separate observations.
inches.					
2·756	35·5	35·398	− 0·102	− ·00288	49·163
2·256	73·0	73·013	+ 0·013	+ ·00018	49·089
1·756	171·25	172·36	+ 1·11	+ ·00644	48·935
1·506	282·45	281·90	− 0·55	− ·00195	49·140
1·256	481·3	483·18	+ 1·88	+ ·00389	48·998
1·006	871·2	866·36	− 4·84	− ·00559	49·230
Mean value of M from the separate observations -					49·093

The quotients in the 5th column are so extremely small, that there can be no doubt of the formula

$$\alpha = \left\{\frac{M}{c^2+\varepsilon^2}\right\}^2,$$

accurately representing the observations in all cases within the above limits of the value of c.

Having clearly ascertained the law of the force by which the magnets urged the copper ring during their rotation under it, I next proposed reversing the experiment by making the copper ring revolve under the same magnets. For this purpose the magnets were placed in the wooden scale, suspended by the wire which in the preceding experiments had

carried the ring: they were placed on their flat sides at the distance ·34 inch from each other, with their poles of the same name adjacent, precisely as they were in the preceding case; and when the index on the scale between their south poles pointed in the magnetic meridian, there was no torsion in the wire. The suspending wire, if produced, would have bisected the line equidistant from the axes of the magnets, and passed through the axis of rotation. The ring was firmly fixed on the top of the frame, G H, so as to revolve horizontally in its own plane, about its centre, under the magnets; and the same precautions as before were taken in order to determine accurately the distance between K and M, corresponding to the zero of distance between the middle section of the ring and the axes of the magnets. I have before stated that the weight suspended in the preceding experiments was 32·56 ounces: in the present case, the weight of the magnets, scale, &c. suspended by the same wire was 33·475 ounces, so that the tension was rather more than before; but this difference was unavoidable, as the weight of the magnets alone is nearly equal to that of the ring.

The effects produced at different distances by the rotation of the ring being measured by the deviation of the magnets from the meridian, in order to compare these effects with those observed when the magnets revolved under the ring, it was necessary to determine the degree of torsion equivalent to any deviation of this compound needle. For this purpose the suspending wire was twisted 2, 4, 6 circles by making the magnets describe 2, 4, 6 circles, first in the direction of screwing, then in the contrary direction, noting the corres-

ponding deviations of the index on the compound needle; and the mean of the deviations in opposite directions being taken, the ratio of the torsion to the sine of the deviation was obtained: the following are the results.

No. of turns of the wire.	Deviation of the index to the magnets.		Mean deviation.	Arc of torsion.	Torsion / Sin. deviation.
	Screwing.	Unscrewing.			
	° ′	° ′	° ′	° ′	
2	6 05 E	6 20 W	6 12½	713 48	6600·465
4	12 30	12 30	12 30	1427 30	6595·386
6	18 55	18 45	18 50	2141 10	6634·320
Mean value of Torsion / sin. deviation					6610·057

The ring was made to revolve with the same velocity as before, viz. 5 revolutions in a second; first in the direction of screwing; and this velocity was carefully maintained until the needle became steady in its direction, when the deviation marked by the index was noted: the same was done in the direction of unscrewing; and the mean of these deviations was taken as the deviation due to the rotation of the ring. When the distance between the middle section of the ring and the axes of the magnets exceeded 2 inches, the deviation was so small, that the errors to which the observation of it was liable, 2′ or 3′, bore too great a proportion to the whole for me to place much reliance on the observations, and I, consequently, have not noted the deviation at the distance 2·756 inches.

In the following table are contained the observed deviations, the torsion equivalent to the force urging the magnets during rotation, and likewise the values of M and ϵ^2 obtained from

the torsions as before: the numbers in the third column are deduced from the observed deviations by substituting them in the expression,

Arc of torsion = 6610. Sin deviation;

and the values of α, or the torsion of the wire which is equivalent to the force with which the ring urged the magnets at different distances, are found by adding the deviation to the preceding arc of torsion.

Table IX.

Distance of the middle section of the ring from the axes of the magnets, or values of c_{II}, c_{III}, &c.	Deviation of the magnets caused by the rotation of the ring.	Torsion of the wire which is equivalent to the deviation.	Torsion of the wire which is equivalent to the force with which the ring urges the magnets, or value of α.	Value of a_n or $\frac{1}{\alpha^3_n}$	Values of M determined from different combinations of a_{II}, a_{III}, a_{IV}, a_V, &c.		Values of ϵ^3 determined from M = 46·63 and the separate observations.
					Combinations.	Values of M.	
					a_{II} a_{IV}	47·828	
					a_{II} a_V	47·497	
					a_{II} a_{VI}	47·525	
					a_{II} a_{VII}	47·075	
c_{II} = 2·256	0°36′	69·2	69·8	a_{II} = ·119694	a_{III} a_V	45·860	·4918
c_{III} = 1·756	1 23·5	160·5	161·9	a_{III} = ·078592	a_{III} a_{VI}	46·347	·5812
c_{IV} = 1·506	2 20	269 1	271·4	a_{IV} = ·060701	a_{III} a_{VII}	45·779	·5624
c_V = 1·256	4 06·5	473·6	477·7	a_V = ·045753	a_{IV} a_{VI}	46·859	·5559
c_{VI} = 1·006	7 30	862·8	870·3	a_{VI} = ·033897	a_{IV} a_{VII}	45·875	·5686
c_{VII} = 0·756	15 27·5	1761·8	1777·3	a_{VII} = ·023720	a_V a_{VII}	45·659	·5345
					Mean values	46·630	·5491

If we compare the torsions in the fourth column, which are equivalent to the force with which the ring by its rotation urges the magnets, with the mean torsions before obtained, which represent the force with which the magnets urge the ring when revolving horizontally under it, we shall find the

agreement such, that there can be no doubt, that the forces are the same at the same distances in the two cases. The mean value of α at the distance ·756 inch when the magnets revolved was 1811·7; but it is to be observed that the value of α in the present instance, 1777·3, should not be compared with this, but with 1763·75, that value of α which is half the arc of torsion when the ring comes to rest, previously to turning in the contrary direction to that of the rotation of the magnets, since in the present case the rotation of the ring was continued until it balanced steadily the directive force of the magnets. This consideration greatly diminishes the difference in the two cases, and will account also in some measure for the values of M and ϵ^3 being different in Table VI. and Table IX.; but I must likewise state that the observations, when the ring revolved under the magnets, were liable to errors which did not affect those in the reverse form of the experiment, although I took great care to determine correctly the direction of the index to the magnets when the force of rotation balanced the directive force.

The comparison between the values of α contained in the preceding table, and those computed for the several values of c from the formula (5), making $M = 46{\cdot}63$ and $\epsilon^3 = {\cdot}5491$, is made in the following table; and their agreement is such, that there can be no doubt, that the effects produced on the magnets by the rotation of the ring at different distances are correctly represented by this formula.

Table X.

Values of c in the different observations.	Observed values of a	Values of a computed from $a=\left\{\frac{M}{c^3+c^3}\right\}^{\frac{1}{3}}$	Diff. between the observed and computed values of a.	Quotient of this diff. divided by the computed value of a.	Values of M deduced from the separate observations.
inches.					
2·256	69·8	68·39	— 1·41	— ·02062	47·108
1·756	161·9	164·78	+ 2·88	+ ·01748	46·221
1·506	271·4	273·98	+ 2·58	+ ·00942	46·410
1·256	477·7	480·79	+ 3·09	+ ·00643	46·480
1·006	870·3	892·21	+ 21·91	+ ·02455	46·054
0·756	1777·3	1731·5	— 45·8	— ·02645	47·242
Mean value of M from the separate observations					46·586

Whether the close agreement between the results deduced from the formulas (1) and (5), and the several observations which I have detailed, or the small differences in the constants deduced from the separate observations in each of the foregoing cases, be considered as a test of the correctness of these formulas, there can, I think, be no doubt that they will very accurately represent the effects that are produced, whether by the rotation of magnets on a copper ring, or on magnets by the rotation of a ring, within certain limits of the distance between the magnets and the ring.

Having succeeded in determining from experiment the law according to which the forces acting during rotation varied at different distances, for which purpose principally I had undertaken these experiments, my next object was to ascertain whether the formula I had obtained would result from the principle, that time is necessary both for the developement of magnetism by induction, and for its dissipation, and which appeared to be implied in the formula.

We have as yet no facts which indicate on what function of the time either the developement or dissipation of magnetism depends; and until this can be ascertained, there appears little prospect of obtaining a complete solution to this interesting but intricate problem in magnetism. It appeared to me not improbable, that when the distance of a magnetic pole from a body capable of becoming magnetic by induction, is suddenly decreased, the magnetism developed in any very small portion of time, varies as the time and the magnetism yet remaining to be developed, before it has attained the maximum intensity of which it is susceptible at this diminished distance, and which may be called the intensity due to this distance; and that on the contrary, on the removal of the pole, the magnetism dissipated varies as the time and the excess of the intensity above the minimum at the increased distance; the rate of developement and dissipation not being however, *cæteris paribus*, necessarily the same. I accordingly assumed this, in order to obtain the intensity of the magnetism in any point of the ring in terms of its distance from the pole of the magnet; but although the differential equation that would result from this assumption is integrable, so that the intensity of the point may be determined, yet as it is in a series whose convergency depends upon the smallness of the angular distance between that point and the point vertically over the pole of the magnet, and a second integration is required to deduce the force with which the magnets urge the ring, the result is rendered so complicated, that to reduce it to any form with which the formula I have obtained from experiment may be compared, appears almost a hopeless task. I consider therefore that, in order to obtain

theoretical results, it is better to adopt in a more general way the principle to which I have referred.

According to this principle, the magnetism in the ring will not be developed with the greatest intensity in the point which is *vertical* to the pole of the magnet at any instant, but in a point at a certain distance *behind* it; and in every point in *advance* of this, and likewise in those *behind*, magnetism will be developed with an intensity depending upon their distances from the pole of the magnet. In the points *behind*, the intensity will be *greater* than, what I have termed, the intensity due to the distance of the magnet; in those in *advance*, *less:* but the same distribution of magnetism in the ring will constantly follow the magnet, during its rotation under it with the same uniform velocity. The effect of the magnetism in the points *behind* the pole of the magnet will be, to urge the ring with a constant force in the *direction of rotation;* that of the magnetism in those in *advance*, to urge it in a *contrary direction*, likewise with a constant force: so that the ring will be urged in the *direction of rotation* with a force which is equivalent to the *excess* of the *former force* above the *latter*. We may therefore assume that the *whole* force of the parts of the ring in *advance* of the magnet will be destroyed by the force of only a *portion* of the part *behind;* and that *the ring is continually urged in the direction of rotation by the undiminished force of the remaining portion of the part behind:* so that to obtain the force of the *whole*, we shall have to integrate from one extremity of *this portion* to the other.

Let r be the radius of the circle described by the poles of the magnets, or the radius of that circle in the ring vertically

over them, and on which I suppose the whole action to take place; s the arc of the latter circle between the point which is vertical to the pole of one of the magnets and the portion ds, whose force is to be determined. Referring the induced magnetism developed in every magnetic particle in ds, to two poles, the force with which these poles act upon the pole of the magnet, and consequently with which this pole acts upon them, will vary directly as the intensity of the magnetism developed, and inversely as the square of the distance: the *south* pole of the magnet, that which in the experiments was nearest to the ring, will *attract* the induced *north* pole, and *repel* the induced *south* pole according to this law; and the axis of polarisation will be in the direction of the line joining the centre of the particle and the pole of the magnet. If we call the intensity of the magnetism developed in each induced pole, i; the distance of each pole from the centre of the particle, k; the distance of the centre of the particle from the pole of the magnet, ρ; then the action of the magnet on the particle in the direction of the line drawn from its centre to that of the magnet, being the difference of the attractive and repulsive forces on the two poles, will be represented by

$$\mu i \left\{ \frac{1}{(\rho - k)^2} - \frac{1}{(\rho + k)^2} \right\}^a ;$$

or by $$\mu i \cdot \frac{4 \rho k}{(\rho^2 - k^2)^2} :$$

or, since k is exceedingly small compared with ρ, by

$$\frac{4 \mu k i}{\rho^3},$$

μ being a constant multiplier.

As this will represent the force on each magnetic particle in ds, the whole force on ds, taking m for the constant multiplier, may be represented by

$$\frac{4 m i ds}{\rho^3};$$

or, if φ be the angular distance of ds from the point vertically over the pole of the magnet, by

$$\frac{4mir\,d\varphi}{\rho^3}.$$

The force on ds, or $r\,d\varphi$, in the direction of a tangent to the ring, that which urges it in the direction of the rotation of the magnet, will therefore be represented by

$$\frac{4mir^2 \sin.\varphi\,d\varphi}{\rho^4}.$$

The intensity i, is, as I have before mentioned, greater than that due to the distance ρ: let us then suppose that it is the intensity corresponding to a position of the magnet at a very small distance ω behind its position at the instant at which we are estimating the force of the magnet on ds. Considering ω as a right line, the distance between this point and ds will be

$$\left\{c^2 + (r \sin.\varphi - \omega)^2 + r^2(1 - \cos.\varphi)^2\right\}^{\frac{1}{2}},$$

or

$$\left\{c^2 + 2r^2(1 - \cos.\varphi) - 2r\omega \sin.\varphi\right\}^{\frac{1}{2}},$$

where c represents the vertical distance between the pole of the magnet and the middle horizontal section of the ring, or $c + p$ in the first set of experiments. We shall therefore have, μ, being a constant multiplier,

$$i = \frac{\mu}{c^2 + 2r^2(1 - \cos.\varphi) - 2r\omega \sin.\varphi};$$

or, since ω is extremely small,

$$i = \frac{\mu}{c^2 + 2r^2(1 - \cos.\varphi)} + \frac{2\mu r\omega \sin.\varphi}{\left\{c^2 + 2r^2(1 - \cos.\varphi)\right\}^2},$$

very nearly.

If then, according to what I have before stated, $r\psi$ represents that portion of the ring, *behind* the magnet, whose action is destroyed by the action of the part in *advance*, the

whole force with which the magnet urges the ring in the direction of its rotation will be represented by

$$2\,M^2 r^2 \cdot \int \left\{ \frac{\text{sin. } \varphi}{\left\{ c'^2 + 2r^2(1-\text{cos. } \varphi) \right\}^{\frac{3}{2}}} + \frac{2r\omega \text{ sin.}^2 \varphi}{\left\{ c'^2 + 2r^2(1-\text{cos. } \varphi) \right\}^2} \right\} \cdot d\varphi,$$

the integral being taken from $\varphi = \psi$ to $\varphi = \pi$, and $\frac{1}{2} M^2$ being put for the constant multiplier $m \mu_{,}$; or if we consider the action of two magnets diametrically opposite to each other, the force will be represented by double this integral taken from $\varphi = \psi$ to $\varphi = \frac{1}{2}\pi$. The force in the latter case will therefore be represented by

$$M^2 \left\{ \frac{1}{\left\{ c'^2 + 2r^2(1-\text{cos. } \psi) \right\}^{\frac{3}{2}}} - \frac{1}{(c'^2 + 2r^2)^{\frac{3}{2}}} \right.$$

$$+ \frac{\text{Sin. } \psi}{c'^2 + 2r^2(1-\text{cos. } \psi)} \cdot \left[\frac{1}{\left\{ c'^2 + 2r^2(1-\text{cos. } \psi) \right\}^{\frac{3}{2}}} - \frac{1}{2c'^2(c'^2+4r^2)} \left\{ \frac{c'^2 + 2r^2}{c'^2 + 2r^2(1-\text{cos. } \psi)} + \frac{c'^4 + 4c'^2r^2 + 12r^4}{c'^2(c'^2+4r^2)} \right\} \right]$$

$$\left. + \frac{4r\omega}{3} \cdot \left\{ - \frac{1}{c'^2 + 2r^2} \left\{ \frac{1}{(c'^2+2r^2)^2} - \frac{c'^4 + 4c'^2r^2 + 6r^4}{c'^4(c'^2+4r^2)^2} \right\} \right.\right.$$

$$\left.\left. + \frac{6r^2(c'^2+2r^2)}{c'^3(c'^2+4r^2)^{\frac{1}{2}}} \left\{ \text{Tan}^{-1} \cdot \frac{\sqrt{(c'^2+4r^2)}}{c'} - \text{Tan}^{-1} \cdot \left(\frac{\sqrt{(c'^2+4r^2)}}{c'} \cdot \text{Tan } \tfrac{1}{2}\psi \right) \right\} \right\}.$$

If ψ be a small arc, ω extremely small, and c' do not exceed $\frac{1}{2} r$, the first term here will greatly exceed any of the others; and the sum of all the terms multiplied by $\frac{4r\omega}{3}$ being plus, this will diminish the second term, which is minus: so that with these limitations we may consider

$$\left\{ \frac{M}{c'^2 + 2r^2(1-\text{cos. } \psi)} \right\}^{\frac{3}{2}}$$

as a very close approximation to the value of the force with which the magnets urge the ring.

In order to obtain a more precise estimate of the value of the terms omitted, let us compare this value of the force with

$$\left\{\frac{M}{c^2+\epsilon^2}\right\}^2,$$

obtained from the experiments.

It appears from the experiments (Tables II. and III.) that ϵ^2 is very nearly $\cdot 15$, and r is nearly $5\cdot 5$, or r^2 nearly 30: so that putting $2r^2(1-\cos.\psi)=\epsilon^2$, we have $\cos.\psi=\cdot 9975$, and $\psi=4^\circ 3'$. In the extreme case, in the experiments in Table I. the distance from the upper surface of the magnets to the middle horizontal section of the ring is $2\cdot 5$ inches, and therefore, from the value afterwards deduced for p, $c'=2\cdot 6$ nearly, which is less than $\frac{1}{2}r$; but supposing $c'=\frac{1}{2}r$, and taking only the first two terms, the expression for the force will be

$$\frac{16M^2}{r^4}\left\{\frac{1}{1\cdot 0404}-\frac{1}{81}\right\}.$$

Here, without even taking into consideration the diminution of the second term by those multiplied by $\frac{4r^2}{3}$, the error arising from the omission of this term in the expression for the force, will not in this, an extreme case, amount to $\frac{1}{80}$ of the whole.

We have obtained the expression for the force with which the magnets urge the ring, on the supposition that the magnets revolve with the same uniform velocity in all cases, considering that velocity as the unit of velocity; but it is evident that for any other velocity, this expression must be multiplied by the velocity. As the value of ψ will also depend upon the velocity, this value may be so considerably increased, that the first term will no longer give an approximate value of the force. Beyond a certain velocity, the value of ψ may increase with the velocity, and we may even conceive the velocity to be so far increased, that ψ becoming $\frac{1}{2}\pi$,

the expression for the force will become equal to 0; and with a still farther increase of the velocity, the force will become negative, and the motion of the ring consequently retrograde. The velocity that would be required to produce this effect with copper, may be much beyond what could be produced with the requisite apparatus; but as the value of ψ, the magnets revolving with the same velocity, must vary considerably with different substances, it is by no means improbable that with steel slightly hardened, or perhaps even with hammered iron, a retrograde motion might be produced by an angular velocity more within our command. The success of this experiment would afford a very striking illustration of the principle which is the basis of the preceding calculation.

With regard to the value of ω, all that we can infer from the experiments is, that it must be extremely small. If $c' = \cdot 6$, then $\omega = \cdot 00005$ would reduce the expression for the force to its first term; and the same would be the case when $c'^3 = 7$, or $c' = 2 \cdot 646$, if $\omega = \cdot 038$. Taking ω to be $\cdot 01$, the error that would arise by neglecting the second and following terms in the expression for the force, considering the first term as 1, would be $- \cdot 0147$, when $c' = \cdot 6$; and it would be $+ \cdot 0084$ when $c' = 2 \cdot 646$, the first term being also considered as 1 in this case. This probably is not far from the real value of ω in the present instance, since if it were much greater than $\cdot 01$, the error that would arise from neglecting it, when c' is small, would be considerable, and would be minus; and although this is the case in the results in Tables VII. and X. it is not so in that in Table III. The errors however by which the observations would be affected from a

small error in the distance, when c' is so small as ·6 or ·7 inch, are considerable; and to this source we ought perhaps to attribute the error in the last observation in Table III. being plus instead of minus, as it ought to be from the omission of the second and following terms.

It appears then, that, within the limits of the values of the distance c', at which the observations were made, ω being extremely small, and ψ about 4°, we may, without any sensible error, omit the second and following terms in the expression for the force, which is thus reduced to the form which represents very accurately the several values of α, which is the measure of that force, in the experiments.

I am aware that it may be objected to the method of investigation which I have adopted, that the value of ψ may not be constant for different values of c', nor that of ω be constant for every point of the ring, behind the portion $r\,\psi$, even for the same value of c': but the variations of these being evidently within very narrow limits, no sensible error can arise from supposing them constant: the perfect coincidence between the formula deduced on this supposition, and that previously obtained from the experiments, proves clearly that this is the case, and that the assumptions which I have made are perfectly admissible.

After the very satisfactory explanation which Mr. Babbage and Mr. Herschel have given of the general phænomena observed during rotation, on the principle that time is requisite both for the developement and for the dissipation of magnetism,* fully to establish the truth of the principle, it remained only to show, that the results obtained from

* Phil. Trans. for 1825.

calculation, founded on that principle, perfectly accord with those obtained from experiment; and this, I trust, I have done by the preceding experimental and theoretical details. Considering this then as an established principle, future investigations must be directed to the discovery of the function of the time on which the intensity of the induced magnetism depends, during the approach of a magnetic pole towards a physical point susceptible of magnetism, and also during its recess. I have stated what I consider to be not an improbable law; but the whole time occupied either in the developement of magnetism by induction, or in its dissipation, is so minute, that it appears extremely difficult to devise experiments that would be a direct test of such laws; and to the more indirect tests, derived from a comparison of such experimental results as the foregoing, with theoretical results derived from these laws, difficulties of analysis in general oppose themselves.

Royal Military Academy,
7th June, 1826.

IX. *Corrections to the reductions of Lieutenant* FOSTER's *Observations on Atmospherical Refractions at Port Bowen; with Addenda to the Table of Magnetic Intensities at the same place. By Lieutenant* HENRY FOSTER, *R. N. F. R. S.*

Read March 22, 1827.

I HASTEN to acquaint the Royal Society with an error I have inadvertently committed in my observations on Atmospherical Refractions at Port Bowen, published in Part IV. of the Philosophical Transactions for 1826. The error alluded to arises from my having used an erroneous value of the divisions on the long level of the repeating circle, with which the observations were made. For the detection of this error, I am indebted to Captain KATER, who recently having had occasion to use the same instrument, found the value of each division of the level to be more than ten seconds in arc, of which he very kindly informed me, knowing that I had considered them as equal to single seconds only; and at the same time he wished me to acquaint him, if any accident had happened to the instrument, by which the level might have been changed. I knew that no accident had befallen the instrument, and that the level was the same which I had used; I therefore immediately proceeded to town, and in Captain KATER's presence ascertained the value of each division of the level to be 10″.9.

In order to explain the source of the error into which I have fallen, I must state, that on receiving the repeating

circle in question, I was given to understand that the value of each division of the level was equal to one second in arc:—and as this instrument was the same that had been employed by Captain SABINE, in his experiments on the pendulum, I took no steps to examine this point for myself, for it never once occurred to me that there could exist a doubt on the subject.

The following, however, are the corrected Tables, which are similarly numbered with those they are intended to be substituted for, in pages 220 and 223 of Part IV. of the Philosophical Transactions, for 1826.

Table VII. page 220, Philosophical Transactions, Part IV. 1826.

Observations for determining the apparent altitude of Arcturus at the time of setting, by Lieutenant FOSTER's *upper* telescope.

The corresponding observations for refraction are contained in Tables VIII. and IX. pages 221 and 222 in Part IV. for 1826.

1825. Day.	Time.	No. of Obs.	Mean Reading of the Four Verniers.	Corrections for Index.	Corrections for Level.	Apparent Altitude.	Barometer at Temp. + 480.	Temp. Fah.t	Winds True.	Weather.	Remarks.
							Inches.				
Feb. 18	At 10h A. M.	16	238 34 17,5	+ 10	+3 13,47	7 35 8,69	29,540	—26	ESE L.	Fine and Clear.	
—— *19	11 A. M.	12	147 31 2,5	—	+1 13,57	7 35 10,12	29,610	—29	Easterly	Hazy, with slight Snow.	
—— 20	10 A. M.	12	56 28 2,5	—	+0 49,05	7 35 10,92	29,626	—35	Calm	Clear and Fine	Thin haze near the horizon.
—— 21	9 A. M. to 1½ P. M.	6	190 55 57,5	—	+0 54,50	7 35 11,75	29,460	—40	Calm	Weather.	Hazy near the horizon.
		6	325 23 57,5	—	+0 10,90	7 35 17,18					
		6	99 52 6,25	—	+0 2,72	7 35 18,09		—36			
Mar. 4	10 A. M.	16	238 34 10,25	0,0	+2 38,05	7 35 11,98	29,600	—31	Calm	Clear and Fine.	

Mean to be used in Table VIII. 7 35 12,68 *instead* of 7 35 18,43
Micrometrical Measure as before + 1 49,82

Altitude to be used in Table IX. = 7 37 2,50 *instead* of 7 37 8,14

* The principal vernier was not reset to zero after the observations on the 18th, but the readings were continued in the subsequent observations, except on March 4th, when the principal vernier was reset to zero.

Table X. page 223; Part IV. for 1826.

Observations for determining the apparent altitude of α Aquilæ at the time of setting, by Lieutenant FOSTER'S *upper* telescope

The corresponding observations for refraction are contained in Tables XI and XII; pages 224 and 225 in Part IV. for 1826.

1825. Day.	Time.	No. of Obs.	Mean readings of four Verniers.	Correction for Index.	Correction for Level.	Apparent zenith distance.	Apparent Altitude deduced from the apparent zenith distance in the ratio of the No. of observations.	Bar. at Temp. + 48°.	Temp. Fah.	Winds true.	Remarks.
	h		° ′ ″	°	′ ″	° ′ ″	° ′ ″	Inches.	°		
Jan. 27	at 11h A.M.	8	323 7 57,5	0,0	+ 1 10,9	85 23 31,05	4 36 22,10	29,970	— 29,5	.....	Occasionally squally.
—	1½ P.M.	14	* 78 38 30	—	+ 1 29,92	85 23 43,03					
Feb. 8	11 A.M.	8	323 8 20	0,0	— 0 5,45	85 23 31,82	4 36 22,02	29,455	— 22,5	North, moderate	
—	2 P.M.	8	* 286 18 5	—	+ 0 8,17	85 23 44,15					
9	10 A.M.	8	323 8 1,25	0,0	+ 0 32,7	85 23 34,24	4 36 19,17	29,701	— 35,7	Calm, fine and clear	
—	1 P.M.	14	* 115 32 16,25	—	+ 0 24,52	85 23 45,77					
10	10½ A.M.	6	* 152 21 51,25	—	+ 0 35,42	85 23 44,44	4 36 20,43	30,100	— 35,5	Calm	
—	1½ P.M.	6	* 304 43 41,25	—	— 0 21,80	85 23 34,70					
15	11 A.M.	12	* 304 43 8,75	—	+ 0 27,25	85 23 38,0	4 36 22,00	29,600	— 33	Cloudy overcast weather.	
28	11 A.M.	6	* 152 22 1,25	—	— 0 38,15	85 23 33,85	4 36 25,19	29,992	— 21	N. E. light, clear, & fine.	
—	1½ P.M.	6	* 304 43 25	—	+ 0 10,9	85 23 35,77					

Mean to be used in Table XI..... = 4° 36′ 21″,82, *instead of* 4° 36′ 23″,08
Micrometrical measure between Telescopes as before + 3 8,42

Altitude to be used in Table XII..... = 4 39 30,24, *instead of* 4 39 31,50

* The principal vernier not reset to zero, and the observations are continued from the preceding reading.

P. S. It is to be regretted, that the observations by Lieutenant Ross, given at the end of the Paper alluded to, must now be wholly rejected; in consequence of the original observations involving the erroneous datum dependent upon the level, having been left on board the Fury at the time of her loss.

The extremely low temperature of the atmosphere, in

which the Observations by Capt. PARRY and myself for the Amount of Refraction at Port Bowen were made, renders it important that the identical thermometer employed should be preserved; I therefore beg to present it to the Society for their acceptance: it was prepared with every possible care by Mr. DANIELL, for the occasion, and is a spirit thermometer, having an ivory scale, graduated from 110° degrees below zero to 94° above, of FAHRENHEIT:—It has also divisions according to REAUMUR's scale.

I avail myself of this opportunity of noticing an omission in the Table of Abstracted Intensities, page 125 of Part IV. of the Philosophical Transactions, for 1826; in which, the means of the times of vibration of the horizontal magnetic needle only are given. The actual intensities would exhibit the law of variation much better than the times, and as I have detected some numerical errors in the Table alluded to, it would be better to recompute the whole, and convert the times into proportional intensities by squaring the reciprocals of the times, and multiplying those squares by 10,000,000,000, in order to render the results all integral. It is now however too late for such recomputation, and all that remains is to put the reader on his guard against the errors alluded to.

HENRY FOSTER.

H. M. S. Hecla, Deptford,
March 15, 1827.

X. *Correction of an error in a Paper published in the Philosophical Transactions, entitled, "On the Parallax of the fixed Stars."* By J. F. W. Herschel, *Esq. F. R. S.*

Read February 22, 1827.

In my Paper on the parallax of the fixed stars, published in the Philosophical Transactions for 1826, by inadvertence the letter l was written for λ, in the formula for computing the most advantageous times of year for observations on the parallax of a double star. The mistake was detected, and rectified in an erratum annexed to the 4th Part of the Transactions for that year, but not till the table computed by the erroneous formula was printed off, and too late to cancel the sheet. The errors produced fall wholly on the 5th column of the table in pages 277, 278, of the volume for that year, headed, "Times of the year most proper for observation;" which column I have therefore had recomputed, and which is here subjoined. I am happy to be able to add, that except in a few instances, the differences are not so considerable as to render useless any observations which may have been made on the plan proposed, as in the great majority of instances the correct dates do not differ by more than a week from the erroneous ones; and about the times proper for observation, a week or a fortnight one way or the other is of no essential moment.

General Number in Messrs. H. and S. Observations.	Star's Name, &c.	Times of the year most proper for Observation.
1	35 Piscium...	June 27, December 27.
20	γ Arietis.....	January 18, July 21.
25	α Piscium....	January 16, July 19.
38	32 Eridani ...	February 13, August 17.
39	ε Persei.....	February 13, August 18.
46	55 Eridani ...	February 6, August 10.
47	ω Aurigæ	March 6, September 8.
53	Rigel	March 20, September 22.
55	118 Tauri ...	March 10, September 13.
56	32 Orionis ...	March 19, September 22.
59	33 Orionis ...	March 22, September 24.
67	ζ Orionis	February 28, September 2.
366	41 Aurigæ ...	March 23, September 26.
69	8 Monocer. ..	April 2, October 5.
76	38 Gemin. ...	March 30, October 3.
80	δ ——	April 6, October 9.
88	11 Cancri	April 18, October 21.
93	φ^3 ——	April 18, October 21.
94	18 BodeHydræ	April 29, November 1.
96	144 of 145 ...	April 21, October 24.
98	57 Cancri....	May 4, November 6.
99	17 Hydræ ...	April 28, October 31.
102	Cancri 194...	April 30, November 2.
114	Leonis 145 ...	May 24, November 25.
128	90 Leonis....	June 5, December 6.
133	65 Ursæ	May 23, November 24.
134	2 Comæ	May 28, November 29.
147	118 of 145 ...	June 3, December 4.
152	Struve 422 ...	June 27, December 27.
155	—— 424...	June 30, December 30.
161	54 Virginis ..	January 14, July 16.
167	81 —— ..	January 12, July 15.
173	98 of 145	January 20, July 23.
176	Struve 456...	January 27, July 30.
177	—— 457...	January 23, July 26.
188	39 Bootis	March 22, September 24.
193	44 ——	June 25, December 25.
194	Struve 474...	January 24, July 27.
201	ν Coronæ B ..	January 22, July 25.
205	∂ Serpentis ...	February 3, August 7.
206	Libræ 178 ...	February 12, August 16.
211	II. 85	March 4, September 6.
212	III. 103	March 10, September 12.
228	g 5 Ophiuchi	February 25, August 30.
240	——	March 1, September 2.
242	μ Draconis ...	February 8, August 12.
245	39 Ophiuchi .	March 8, September 11.
262	100 Herculis .	March 19, September 21.
265	I. 86	March 29, October 2.
269	39 Draconis ..	March 22, September 24.
271	——	April 15, October 18.
274	——	April 3, October 7.
280	——	March 28, October 1.
287	——	April 14, October 18.
295	III. 57	April 27, October 30.
306	π Aquilæ	May 10, November 12.
311	ι Draconis ...	April 20, October 23.
312	↓ Cygni.....	April 18, October 21.
313	I. 96.....	April 20, October 23.
317	II. 96	April 6, September 10.
320	I. 95	May 12, November 14.
323	ε Capric.	April 23, October 26.
326	——	April 27, October 30.
343	Struve 751 ...	February 14, August 18.
349	Aquarii 213 ..	May 28, November 29.
352	——	June 24, December 24.
354	94 Aquarii...	May 21, November 22.
356	107 —— ...	June 4, December 6.
359	σ Capric.	July 16, January 14.